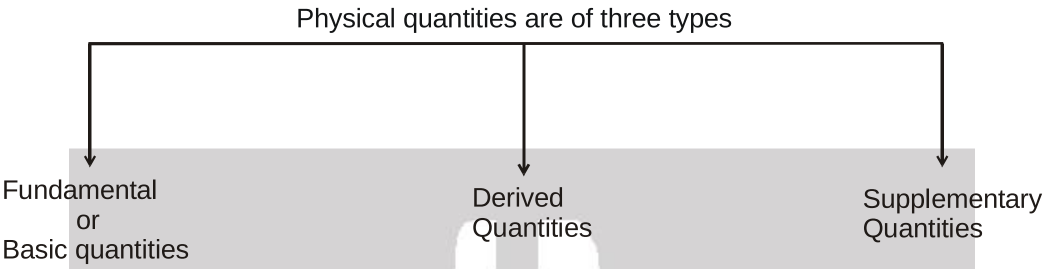
UNITS & DIMENSIONS

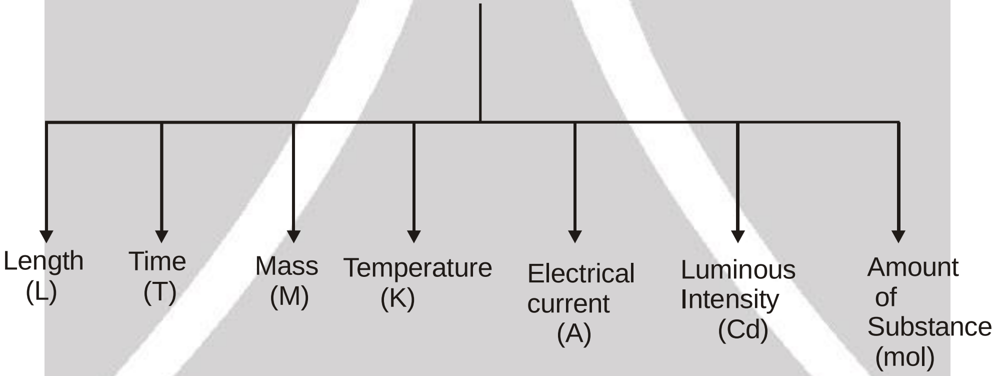
I. PHYSICAL QUANTITIES :

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities. Till class X we have studied many physical quantities eg. length, velocity, acceleration, force, time, pressure, mass, density etc.

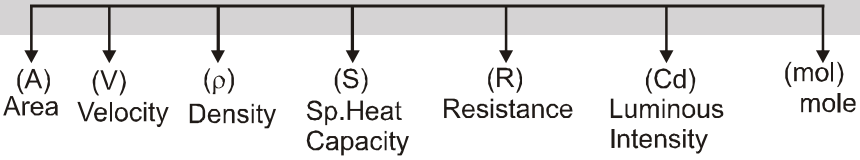


1. Fundamental (Basic) Quantities :

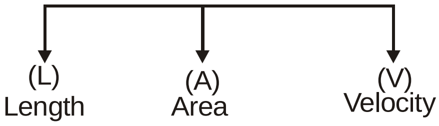
- These are the elementary quantities which covers the entire span of physics.
- Any other quantities can be derived from these.
- All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance (d), time (t) and velocity (v) cannot be chosen as basic quantities (because they are related as $V = \frac{d}{t}$). An International Organization named CGPM : General Conference on weight and Measures, chose seven physical quantities as basic or fundamental.



These are the elementary quantities (in our planet) that's why chosen as basic quantities.
In fact any set of independent quantities can be chosen as basic quantities by which all other physical quantities can be derived.
i.e.,



Can be chosen as basic quantities (on some other planet, these might also be used as basic quantities)
But



cannot be used as basic quantities as
Area = (Length)^2 so they are not independent.

2. Derived Quantities :

Physical quantities which can be expressed in terms of basic quantities (M,L,T....) are called derived quantities.

i.e., Momentum P = mv

$$= (m)\ \frac{\text{displacement}}{\text{time}} = \frac{ML}{T} = M^1\ L^1\ T^{-1}$$

Here $[M^1L^1T^{-1}]$ is called dimensional formula of momentum, and we can say that momentum has

1 Dimension in M (mass)

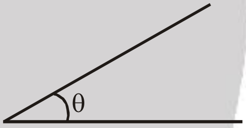
1 Dimension in L (length)

and –1 Dimension in T (time)

The representation of any quantity in terms of basic quantities (M, L, T....) is called dimensional formula and in the representation, the powers of the basic quantities are called dimensions.

3. Supplementary quantities :

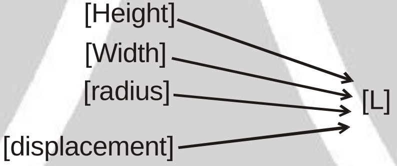
Besides seven fundamental quantities two supplementary quantities are also defined. They are



- Plane angle (The angle between two lines)
- Solid angle

II. FINDING DIMENSIONS OF VARIOUS PHYSICAL QUANTITIES :

- Height, width, radius, displacement etc. are a kind of length. So we can say that their dimension is [L]



  here [Height] can be read as "Dimension of Height"

- Area = Length × Width

  So, dimension of area is [Area] = [Length] × [Width]

  $= [L] \times [L] = [L^2]$

  For circle

  Area = πr^2

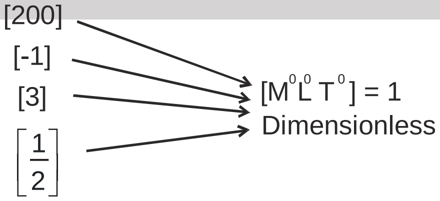
  $[\text{Area}] = [\pi]\ [r^2]$

  $= [1]\ [L^2]$

  $= [L^2]$

  Here π is not a kind of length or mass or time so π shouldn't affect the dimension of Area.

  Hence its dimension should be 1 $(M^0L^0T^0)$ and we can say that it is dimensionless. From similar logic we can say that all the numbers are dimensionless.



- $[\text{Volume}] = [\text{Length}] \times [\text{Width}] \times [\text{Height}] = L \times L \times L = [L^3]$

  For sphere

  $$\text{Volume} = \frac{4}{3}\pi r^3$$

  $$[\text{Volume}] = \left[\frac{4}{3}\pi\right]\ [r^3] = (1)\ [L^3] = [L^3]$$

  So dimension of volume will be always $[L^3]$ whether it is volume of a cuboid or volume of sphere.

Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.

- Density = $\frac{\text{mass}}{\text{volume}}$

  [Density] = $\frac{[\text{mass}]}{[\text{volume}]} = \frac{M}{L^3} = [M^1L^{-3}]$

- Velocity (v) = $\frac{\text{displacement}}{\text{time}}$

  $[v] = \frac{[\text{Displacement}]}{[\text{time}]} = \frac{L}{T} = [M^0L^1T^{-1}]$

- Acceleration (a) = $\frac{dv}{dt}$

  $[a] = \frac{dv \rightarrow \text{kind of velocity}}{dt \rightarrow \text{kind of time}} = \frac{LT^{-1}}{T} = LT^{-2}$

- Momentum (P) = mv

  $[P] = [M]\,[v]$

  $= [M]\,[LT^{-1}]$

  $= [M^1L^1T^{-1}]$

- Force (F) = ma

  $[F] = [m]\,[a]$

  $= [M]\,[LT^{-2}]$

  $= [M^1L^1T^{-2}]$(You should remember the dimensions of force because it is used several times)

- Work or Energy = force × displacement

  [Work] = [force] [displacement]

  $= [M^1L^1T^{-2}]\,[L]$

  $= [M^1L^2T^{-2}]$

- Power = $\frac{\text{work}}{\text{time}}$

  [Power] = $\frac{[\text{work}]}{[\text{time}]} = \frac{M^1L^2T^{-2}}{T} = [M^1L^2T^{-3}]$

- Pressure = $\frac{\text{Force}}{\text{Area}}$

  [Pressure] = $\frac{[\text{Force}]}{[\text{Area}]} = \frac{M^1L^1T^{-2}}{L^2} = M^1L^{-1}T^{-2}$

1. Dimensions of angular quantities :

- Angle (θ)

  (Angular displacement) $\theta = \frac{\text{Arc}}{\text{radius}}$

  $[\theta] = \frac{[\text{Arc}]}{[\text{radius}]} = \frac{L}{L} = [M^0L^0T^0]$ (Dimensionless)

- Angular velocity $(\omega) = \frac{\theta}{t}$; $[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = [M^0L^0T^{-1}]$

- Angular acceleration $(\alpha) = \frac{d\omega}{dt}$; $[\alpha] = \frac{[d\omega]}{[dt]} = \frac{M^0L^0T^{-1}}{T} = [M^0L^0T^{-2}]$

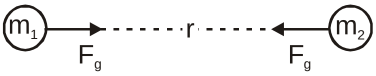
- Torque = Force × Arm length

  [Torque] = [force] × [arm length]

  $= [M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$

2. Dimensions of Physical Constants :

● Gravitational Constant :



If two bodies of mass m_1 and m_2 are placed at r distance, both feel gravitational attraction force, whose value is,

Gravitational force $F_g = \frac{Gm_1m_2}{r^2}$

where G is a constant called Gravitational constant.

$[F_g] = \frac{[G][m_1][m_2]}{[r^2]}$

$[M^1L^1T^{-2}] = \frac{[G][M][M]}{[L^2]}$

$[G] = M^{-1}\,L^3\,T^{-2}$

● Specific heat capacity :

To increase the temperature of a body by ΔT, Heat required is $Q = ms\,\Delta T$

Here s is called specific heat capacity.

$[Q] = [m]\,[s]\,[\Delta T]$

Here Q is heat : A kind of energy so $[Q] = M^1L^2T^{-2}$

$[M^1L^2T^{-2}] = [M]\,[s]\,[K]$

$[s] = [M^0L^2T^{-2}K^{-1}]$

● Gas constant (R) :

For an ideal gas, relation between pressure (P)
Value (V) , Temperature (T) and moles of gas (n) is
PV = nRT where R is a constant, called gas constant.

$[P]\,[V] = [n]\,[R]\,[T]$ .....(1)

here $[P]\,[V] = \frac{[\text{Force}]}{[\text{Area}]}$ $[\text{Area} \times \text{Length}] = [\text{Force}] \times [\text{Length}] = [M^1L^1T^{-2}]\,[L^1] = M^1L^2T^{-2}$

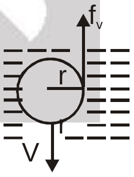
From equation (1)

$[P]\,[V] = [n]\,[R]\,[T]$

$\Rightarrow [M^1L^2T^{-2}] = [\text{mol}]\,[R]\,[K] \quad \Rightarrow \quad [R] = [M^1L^2T^{-2}\,\text{mol}^{-1}\,K^{-1}]$

● Coefficient of viscosity :

If any spherical ball of radius r moves with velocity v in a viscous liquid, then viscous force acting on it is given by



$F_v = 6\pi\eta rv$

Here η is coefficient of viscosity

$[F_v] = [6\pi]\,[\eta]\,[r]\,[v]$

$M^1L^1T^{-2} = (1)\,[\eta]\,[L]\,[LT^{-1}]$

$[\eta] = M^1L^{-1}T^{-1}$

● Planck's constant :

If light of frequency υ is falling , energy of a photon is given by

$E = h\upsilon$ Here h = Planck's constant

$[E] = [h]\,[\upsilon]$

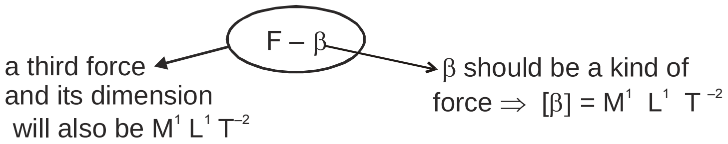
$\upsilon = \text{frequency} = \frac{1}{\text{Time} \quad \text{Period}} \quad \Rightarrow \quad [\upsilon] = \frac{1}{[\text{Time} \quad \text{Period}]} = \left[\frac{1}{T}\right]$

so $M^1L^2T^{-2} = [h]\,[T^{-1}]$

$[h] = M^1L^2T^{-1}$

3. Some special features of dimensions :

- Suppose in any formula, (L + α) term is coming (where L is length). As length can be added only with a length, so α should also be a kind of length.
  So $[\alpha] = [L]$
- Similarly consider a term (F – β) where F is force. A force can be added/subtracted with a force only and give rises to a third force. So β should be a kind of force and its result (F – β) should also be a kind of force.



Rule No. 1 : One quantity can be added / subtracted with a similar quantity only and give rise to the similar quantity.

Solved Example

Example 1. $\frac{\alpha}{t^2} = Fv + \frac{\beta}{x^2}$. Find dimensional formula for $[\alpha]$ and $[\beta]$ (here t = time, F = force, v = velocity, x = distance)

Solution : Since dimension of Fv = $[Fv] = [M^1L^1T^{-2}]\ [L^1T^{-1}] = [M^1L^2T^{-3}]$,

so $\left[\frac{\beta}{x^2}\right]$ should also be $M^1L^2T^{-3}$

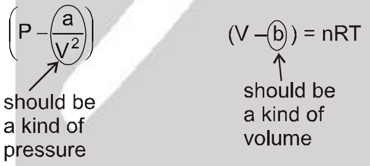
$\frac{[\beta]}{[x^2]} = M^1\,L^2T^{-3}$; $[\beta] = M^1L^4T^{-3}$

and $\left[Fv + \frac{\beta}{x^2}\right]$ will also have dimension $M^1L^2T^{-3}$, so L.H.S. should also have the same

dimension $M^1L^2T^{-3}$ so $\frac{[\alpha]}{[t^2]} = M^1L^2T^{-3}$ $[\alpha] = M^1L^2T^{-1}$

Example 2. For n moles of gas, Vander waal's equation is $\left(P - \frac{a}{V^2}\right)$ (V – b) = nRT. Find the dimensions of a and b, where P is gas pressure, V = volume of gas T = temperature of gas

Solution :



So $\frac{[a]}{[V^2]} = M^1L^{-1}T^{-2}$ So $[b] = L^3$

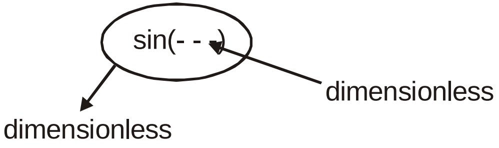
$\frac{[a]}{[L^3]^2} = M^{-1}\,L^{-1}\,T^{-2}$ ⇒ $[a] = M^1\,L^5\,T^{-2}$

Rule No. 2 : Consider a term sin(θ)

Here θ is dimensionless and sinθ $\left(\frac{\text{Perpendicular}}{\text{Hypoteneous}}\right)$ is also dimensionless.

⇒ Whatever comes in sin(......) is dimensionless and entire [sin (.......)] is also dimensionless.

⇒



Similarly :

$\cos(- - -)$
dimensionless
dimensionless

$\tan(- - -)$
dimensionless
dimensionless

$2^{(- - -)}$
dimensionless
dimensionless

$e^{(- - -)}$
dimensionless
dimensionless

$\log_e(- - -)$
dimensionless
dimensionless

Solved Example

Example 3. $\alpha = \frac{F}{v^2}\sin(\beta t)$(here v = velocity, F = force, t = time). Find the dimension of α and β

Solution : $\alpha = \frac{F}{v^2}\sin(\beta t)$

dimensionless
dimensionless

$\Rightarrow [\beta]\ [t] = 1$

$[\beta] = [T^{-1}]$

So $[\alpha] = \frac{[F]}{[v^2]} = \frac{[M^1L^1T^{-2}]}{[L^1T^{-1}]^2} = M^1L^{-1}T^0$

Example 4. $\alpha = \frac{Fv^2}{\beta^2}\log_e\left(\frac{2\pi\beta}{v^2}\right)$ where F = force , v = velocity. Find the dimensions of α and β.

Solution : $\alpha = \frac{Fv^2}{\beta^2}\log_e\frac{2\pi\beta}{v^2}$

dimensionless
dimensionless

$$\Rightarrow \frac{[2\pi][\beta]}{[v^2]} = 1$$

$$\Rightarrow \frac{[1][\beta]}{L^2T^{-2}} = 1 \qquad \Rightarrow [\beta] = L^2T^{-2}$$

$$\text{as } [\alpha] = \frac{[F][v^2]}{[\beta^2]} \qquad \Rightarrow [\alpha] = \frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[L^2T^{-2}]^2} \qquad \Rightarrow [\alpha] = M^1L^{-1}T^0$$

4. USES OF DIMENSIONS :

- **To check the correctness of the formula :**
  If the dimensions of the L.H.S and R.H.S are same, then we can say that this eqn. is at least dimensionally correct. So this equation may be correct.

But if dimensions of L.H.S and R.H.S is not same then the equation is not even dimensionally correct.
So it cannot be correct.

e.g. A formula is given centrifugal force $F_e = \frac{mv^2}{r}$

(where m = mass , v = velocity , r = radius)
we have to check whether it is correct or not.
Dimension of L.H.S is

$[F] = [M^1L^1T^{-2}]$

Dimension of R.H.S is $\frac{[m]\,[v^2]}{[r]} = \frac{[M]\,[LT^{-1}]^2}{[L]} = [M^1L^1T^{-2}]$

So this eqn. is at least dimensionally correct.

Thus we can say that this equation may be correct.

Solved Example

Example 5. Check whether this equation may be correct or not.

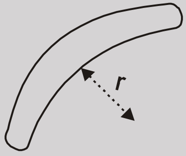
Solution : Pressure $P_r = \frac{3\,Fv^2}{\pi^2t^2x}$ (where P_r = Pressure , F = force , v = velocity , t = time , x = distance)

Dimension of L.H.S = $[P_r] = M^1L^{-1}T^{-2}$

Dimension of R.H.S = $\frac{[3]\,[F]\,[v^2]}{[\pi]\,[t^2]\,[x]} = \frac{[M^1L^1T^{-2}]\,[L^2T^{-2}]}{[T^2]\,[L]} = M^1L^2T^{-6}$

Dimension of L.H.S and R.H.S are not same. So the relation cannot be correct.
Sometimes a question is asked which is beyond our syllabus, then certainly it must be the question of dimensional analyses.

Example 6. A Boomerang has mass m surface Area A, radius of curvature of lower surface = r and it is moving with velocity v in air of density ρ. The resistive force on it can be –



(A) $\frac{2\rho vA}{r^2}\log\left(\frac{\rho m}{\pi Ar}\right)$ (B) $\frac{2\rho v^2A}{r}\log\left(\frac{\rho A}{\pi m}\right)$ (C) $2\rho v^2A\log\left(\frac{\rho Ar}{\pi m}\right)$ (D) $\frac{2\rho v^2A}{r^2}\log\left(\frac{\rho Ar}{\pi m}\right)$

Answer : **(C)**

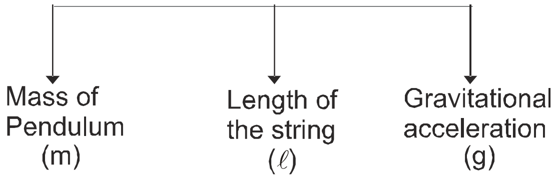
Solution : Only C is dimensionally correct.

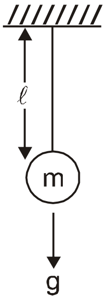
- **We can derive a new formula roughly :**

If a quantity depends on many parameters, we can estimate, to what extent, the quantity depends on the given parameters !

Solved Example

Example 7. Time period of a simple pendulum can depend on





So we can say that expression of T should be in this form

T = (Some Number) $(m)^a (\ell)^b (g)^c$

Equating the dimensions of LHS and RHS,

$M^0L^0T^1 = (1)\,[M^1]^a\,[L^1]^b\,[L^1T^{-2}]^c$

$M^0L^0T^1 = M^a\,L^{b+c}\,T^{-2c}$

Comparing the powers of M,L and T,

get $a = 0$, $b + c = 0$, $-2c = 1$

so $a = 0$, $b = \frac{1}{2}$, $c = -\frac{1}{2}$ so T = (some Number) $M^0\,L^{1/2}\,g^{-1/2}$

T = (Some Number) $\sqrt{\frac{\ell}{g}}$

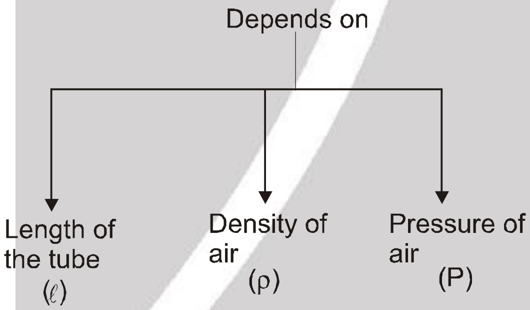
The quantity “Some number” can be found experimentally. Measure the length of a pendulum and oscillate it, find its time period by stopwatch.

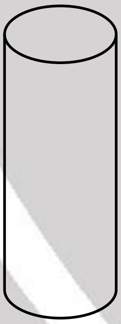
Suppose for ℓ = 1m, we get T = 2 sec. so

2 = (Some Number) $\sqrt{\frac{1}{9.8}}$

$\Rightarrow$ “Some number” = 6.28 $\approx 2\pi$.

Example 8. Natural frequency (f) of a closed pipe





So we can say that f = (some Number) $(\ell)^a (\rho)^b (P)^c$

$\left[\frac{1}{T}\right] = (1)\,[L]^a\,[ML^{-3}]^b\,[M^1L^{-1}T^{-2}]^c$

$M^0L^0T^{-1} = M^{b+c}\,L^{a-3b-c}\,T^{-2c}$

comparing powers of M, L, T

$0 = b + c$

$0 = a - 3b - c$

$-1 = -2c$

get $a = -1$, $b = -1/2$, $c = 1/2$

So f = (some number) $\frac{1}{\ell}\sqrt{\frac{P}{\rho}}$

- **We can express any quantity in terms of the given basic quantities.**

Solved Example

Example 9. If velocity (V), force (F) and time (T) are chosen as fundamental quantities, express (i) mass and (ii) energy in terms of V, F and T

Solution : Let $M = (\text{some Number})\ (V)^a\ (F)^b\ (T)^c$

Equating dimensions of both the sides

$M^1L^0T^0 = (1)\ [L^1T^{-1}]^a\ [M^1L^1T^{-2}]^b\ [T^1]^c$

$M^1L^0T^0 = M^bL^{a+b}\ T^{-a-2b+c}$

get $a = -1,\ b = 1,\ c = 1$

$M = (\text{Some Number})\ (V^{-1}\ F^1\ T^1) \Rightarrow \qquad [M] = [V^{-1}\ F^1\ T^1]$

Similarly we can also express energy in terms of V, F, T

Let $[E] = [\text{some Number}]\ [V]^a\ [F]^b\ [T]^c$

$\Rightarrow\ [ML^2T^{-2}] = [M^oL^oT^o]\ [LT^{-1}]^a\ [MLT^{-2}]^b\ [T]^c$

$\Rightarrow\ [M^1L^2T^{-2}] = [M^b\ L^{a+b}\ T^{-a-2b+c}]$

$\Rightarrow\ 1 = b;\ 2 = a + b\ ;\ -2 = -a - 2b + c$

get $a = 1\ ;\ b = 1\ ;\ c = 1$

$\therefore\ E = (\text{some Number})\ V^1F^1T^1$ or $[E] = [V^1][F^1][T^1]$.

- **To find out unit of a physical quantity :**

Suppose we want to find the unit of force. We have studied that the dimension of force is

$[\text{Force}] = [M^1L^1T^{-2}]$

As unit of M is kilogram (kg) , unit of L is meter (m) and unit of T is second (s) so unit of force can be written as $(kg)^1\ (m)^1\ (s)^{-2} = kg\ m/s^2$ in MKS system. In CGS system, unit of force can be written as $(g)^1\ (cm)^1\ (s)^{-2} = g\ cm/s^2$.

III. LIMITATIONS OF DIMENSIONAL ANALYSIS :

From Dimensional analysis we get $\qquad T = (\text{Some Number})\ \sqrt{\frac{\ell}{g}}$

so the expression of T can be

$T = 2\sqrt{\frac{\ell}{g}} \qquad T = \sqrt{\frac{\ell}{g}}\ \sin(.....)$

or $\qquad$ or

$T = 50\sqrt{\frac{\ell}{g}} \qquad T = \sqrt{\frac{\ell}{g}}\ \log(......)$

or $\qquad$ or

$T = 2\pi\sqrt{\frac{\ell}{g}} \qquad T = \sqrt{\frac{\ell}{g}} + (t_0)$

- Dimensional analysis doesn't give information about the "some Number": The dimensional constant.
- This method is useful only when a physical quantity depends on other quantities by multiplication and power relations.
  (i.e., $f = x^a y^b z^c$)
  It fails if a physical quantity depends on sum or difference of two quantities
  (i.e.$f = x + y - z$)
  i.e., we cannot get the relation
  $S = ut + \frac{1}{2}at^2$ from dimensional analysis.
- This method will not work if a quantity depends on another quantity as sine or cosine, logarithmic or exponential relation. The method works only if the dependence is by power functions.
- We equate the powers of M,L and T hence we get only three equations. So we can have only three variable (only three dependent quantities)

  So dimensional analysis will work only if the quantity depends only on three parameters, not more than that.

Solved Example

Example 10. Can Pressure (P), density (ρ) and velocity (v) be taken as fundamental quantities ?

Solution : P, ρ and v are not independent, they can be related as $P = \rho v^2$, so they cannot be taken as fundamental variables.

To check whether the 'P', 'ρ', and 'V' are dependent or not, we can also use the following mathematical method :

$[P] = [M^1L^{-1}T^{-2}]$
$[\rho] = [M^1L^{-3}T^0]$
$[V] = [M^0L^1T^{-1}]$

Check the determinant of their powers : $\begin{vmatrix} 1 & -1 & -2 \\ 1 & -3 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 1 (3) - (-1)(-1) - 2 (1) = 0,$

So these three terms are dependent.

DIMENSIONS BY SOME STANDARD FORMULAE :-

In many cases, dimensions of some standard expression are asked

e.g. find the dimension of ($\mu_0\varepsilon_0$)

for this, we can find dimensions of μ_0 and ε_0, and multiply them, but it will be very lengthy process. Instead of this, we should just search a formula, where this term ($\mu_0\varepsilon_0$) comes.

It comes in $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ (where c = speed of light)

$\Rightarrow \mu_0 \varepsilon_0 = \frac{1}{c^2}$

$[\mu_0\varepsilon_0] = \frac{1}{c^2} = \frac{1}{(L/T)^2} = L^{-2}T^2$

Solved Example

Example 11. Find the dimensions of

(i) $\varepsilon_0 E^2$ (ε_0 = permittivity in vacuum, E = electric field)

(ii) $\frac{B^2}{\mu_0}$ (B = Magnetic field, μ_0 = magnetic permeability)

(iii) $\frac{1}{\sqrt{LC}}$ (L = Inductance, C = Capacitance)

(iv) RC (R = Resistance, C = Capacitance)

(v) $\frac{L}{R}$ (R = Resistance, L = Inductance)

(vi) $\frac{E}{B}$ (E = Electric field, B = Magnetic field)

(vii) $G\varepsilon_0$ (G = Universal Gravitational constant, ε_0 = permittivity in vacuum)

(viii) $\frac{\phi_e}{\phi_m}$ (ϕ_e = Electrical flux; ϕ_m = Magnetic flux)

Solution : (i) Energy density =1/2 $\varepsilon_0 E^2$

[Energy density] = $[\varepsilon_0 E^2]$

$$\left[\frac{1}{2}\varepsilon_0 E^2\right] = \frac{[\text{energy}]}{[\text{volume}]} = \frac{M^1L^2T^{-2}}{L^3} = M^1L^{-1}T^{-2}$$

(ii) $\frac{1}{2}\frac{B^2}{\mu_0}$ = Magnetic energy density

$$\left[\frac{1}{2}\frac{B^2}{\mu_0}\right] = [\text{Magnetic Energy density}]$$

$$\left[\frac{B^2}{\mu_0}\right] = \frac{[\text{energy}]}{[\text{volume}]} = \frac{M^1L^2T^{-2}}{L^3} = M^1L^{-1}T^{-2}$$

(iii) $\frac{1}{\sqrt{LC}}$ = angular frequency of L – C oscillation

$$\left[\frac{1}{\sqrt{LC}}\right] = [\omega] = \frac{1}{T} = T^{-1}$$

(iv) RC = Time constant of RC circuit = a kind of time

[RC] = [time] = T^1

(v) $\frac{L}{R}$ = Time constant of L – R circuit

$$\left[\frac{L}{R}\right] = [\text{time}] = T^1$$

(vi) magnetic force $F_m = qvB$, electric force $F_e = qE$

$\Rightarrow$ $[F_m] = [F_e]$ $\Rightarrow$ $[qvB] = [qE]$ $\Rightarrow$ $\left[\frac{E}{B}\right] = [v] = LT^{-1}$

(vii) Gravitational force $F_g = \frac{Gm^2}{r^2}$, Electrostatic force $F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

$$\left[\frac{Gm^2}{r^2}\right] = \left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right] \;;\; [G\varepsilon_0] = \left[\frac{q^2}{m^2}\right] = \left[\frac{(it)^2}{m^2}\right] = A^2T^2M^{-2}$$

(viii) $\left[\frac{\phi_e}{\phi_m}\right] = \left[\frac{ES}{BS}\right] = \left[\frac{E}{B}\right] = [\mathbf{v}]$ (from part (vi)) $= LT^{-1}$

Dimensions of quantities related to Electromagnetic and Heat (only for XII and XIII students)

(i) Charge (q) : We know that electrical current $i = \frac{dq}{dt} = \frac{\text{a small charge flow}}{\text{small time interval}}$

$[i] = \frac{[dq]}{[dt]}$; $[A] = \frac{[q]}{t}$ $\Rightarrow$ $[q] = [A^1T^1]$

(ii) Permittivity in Vacuum (ε_0) : Electrostatic force between two charges $F_e = \frac{kq_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

$[F_e] = \frac{1}{[4\pi][\varepsilon_0]}\frac{[q_1][q_2]}{[r]^2}$

$M^1L^1T^{-2} = \frac{1}{(1)[\varepsilon_0]}\frac{[AT][AT]}{[L]^2}$; $[\varepsilon_0] = M^{-1}L^{-3}T^4A^2$

(iii) Electric Field (E) : Electrical force per unit charge $E = F/q$

$[E] = \frac{[F]}{[q]} = \frac{[M^1L^1T^{-2}]}{[A^1T^1]} = M^1L^1T^{-3}A^{-1}$

(iv) Electrical Potential (V) : Electrical potential energy per unit charge $V = U/q$

$[V] = \frac{[U]}{[q]} = \frac{[M^1L^2T^{-2}]}{[A^1T^1]} = M^1L^2T^{-3}A^{-1}$

(v) Resistance (R) : From Ohm's law $V = iR$
$[V] = [i]\,[R]$
$[M^1L^2T^{-3}A^{-1}] = [A^1]\,[R]$; $[R] = M^1L^2T^{-3}A^{-2}$

(vi) Capacitance (C) : $C = \frac{q}{V} \Rightarrow [C] = \frac{[q]}{[V]} = \frac{[A^1T^1]}{[M^1L^2T^{-3}A^{-1}]}$

$[C] = M^{-1}L^{-2}T^4A^2$

(vii) Magnetic field (B) : magnetic force on a current carrying wire $F_m = i\,\ell B \Rightarrow [F_m] = [i]\,[\ell]\,[B]$
$[M^1L^1T^{-2}] = [A^1]\,[L^1]\,[B]$; $[B] = M^1L^OT^{-2}A^{-1}$

(viii) Magnetic permeability in vacuum (μ_0) : Force/length between two wires $\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{i_1i_2}{r}$

$\frac{M^1L^1T^{-2}}{L^1} = \frac{[\mu_O]}{[2\pi]}\frac{[A][A]}{[L]}$ $\Rightarrow$ $[\mu_0] = M^1L^1T^{-2}A^{-2}$

(ix) Inductance (L) : Magnetic potential energy stored in an inductor $U = 1/2\,Li^2$
$[U] = [1/2]\,[L]\,[i]^2$
$[M^1L^2T^{-2}] = (1)\,[L]\,(A)^2$
$[L] = M^1L^2T^{-2}A^{-2}$

(x) Thermal Conductivity : Rate of heat flow through a conductor $\frac{dQ}{dt} = \mathbf{K}A\left(\frac{dT}{dx}\right)$

$\frac{[dQ]}{[dt]} = [\mathbf{k}]\,[A]\frac{[dT]}{[dx]}$; $\frac{[M^1L^2T^{-2}]}{[T]} = [\mathbf{K}]\,[L^2]\,\frac{[K]}{[L^1]}$; $[\mathbf{K}] = M^1L^1T^{-3}K^{-1}$

(xi) Stefan's Constant (σ) : If a black body has temperature (T), then Rate of radiation energy emitted

$\frac{dE}{dt} = \sigma A T^4$; $\frac{[dE]}{[dt]} = [\sigma]\,[A]\,[T^4]$

$\frac{[M^1L^2T^{-2}]}{[T]} = [\sigma]\,[L^2]\,[K^4]$; $[\sigma] = [M^1L^0T^{-3}K^{-4}]$

(xii) Wien's Constant : Wavelength corresponding to max. spectral intensity. $\lambda_m = b/T$ (where T = temp. of the black body)

$[\lambda_m] = \frac{[b]}{[T]}$; $[L] = \frac{[b]}{[K]}$ $[b] = [L^1K^1]$

UNIT

- **Unit :** Measurement of any physical quantity is expressed in terms of an internationally accepted certain basic standard called unit.
- **SI Units :** In 1971, an international Organization "CGPM" : (General Conference on weight and Measure) decided the standard units, which are internationally accepted. These units are called SI units (International system of units)

1. SI Units of Basic Quantities :

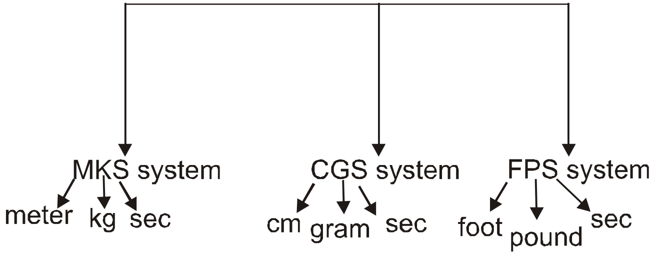
Base Quantity	SI Units		
	Name	**Symbol**	**Definition**
Length	metre	m	The metre is the length of the path traveled by light in vacuum during a time interval of 1/299, 792, 458 of a second (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967)
Electric Current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, will produce between these conductors a force equal to 2 x 10^{-7} Newton per metre of length. (1948)
Thermodynamic Temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of Substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous Intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 x 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979).

2. Two supplementary units were also defined :

- Plane angle – Unit = radian (rad)
- Solid angle – Unit = Steradian (sr)

3. Other classification :

If a quantity involves only length, mass and time (quantities in mechanics), then its unit can be written in MKS, CGS or FPS system.



- **For MKS system :**
  In this system Length, mass and time are expressed in meter, kg and second. respectively. It comes under SI system.
- **For CGS system :**
  In this system ,Length, mass and time are expressed in cm, gram and second. respectively.
- **For FPS system :**
  In this system, length, mass and time are measured in foot, pound and second. respectively.

4. SI units of derived Quantities :

- Velocity = $\frac{\text{displacement (metre)}}{\text{time (second)}}$
  So unit of velocity will be m/s
- Acceleration = $\frac{\text{change in velocity}}{\text{time}} = \frac{m/s}{s} = \frac{m}{s^2}$
- Momentum = mv, so unit of momentum will be = (kg) (m/s) = kg m/s
- Force = ma, Unit will be = (kg) × (m/s^2) = kg m/s^2 called newton (N)
- Work = FS, unit = (N) × (m) = N m called joule (J)
- Power = $\frac{\text{work}}{\text{time}}$, Unit = J / s called watt (W)

5. Units of some physical Constants :

- Unit of "Universal Gravitational Constant" (G)

$$F = \frac{G(m_1)(m_2)}{r^2} \quad \Rightarrow \frac{kg \times m}{s^2} = \frac{G(kg)(kg)}{m^2} \quad \text{so unit of G} = \frac{m^3}{kg\ s^2}$$

- **Unit of specific heat capacity (s) :** Q = ms ΔT ; J = (kg) (S) (K), Unit of s = J / kg K
- **Unit of μ_0 :** force per unit length between two long parallel wires is: $\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$

$$\frac{N}{m} = \frac{\mu_0}{(1)}\frac{(A)\ (A)}{(m)} \qquad \text{Unit of } \mu_0 = \frac{N}{A^2}$$

6. SI Prefix : Suppose distance between kota to Jaipur is 3000 m. so

$$d = 3000\text{ m} = 3 \times \underbrace{1000}_{\text{kilo(k)}}\text{m}$$

= 3 km (here 'k' is the prefix used for 1000 (10^3))

Suppose thickness of a wire is 0.05 m

$$d = 0.05\text{ m} = 5 \times \underbrace{10^{-2}}_{\text{centi(c)}}\text{m}$$

= 5 cm (here 'c' is the prefix used for (10^{-2}))

Similarly, the magnitude of physical quantities vary over a wide range. So in order to express the very large magnitude as well as very small magnitude more compactly, "CGPM" recommended some standard prefixes for certain power of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	K	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

Solved Example

Example 12. Convert all in meters (m) :

(i) 5 μm. (ii) 3 km (iii) 20 mm (iv) 73 pm (v) 7.5 nm

Solution :
(i) 5 μm = 5×10^{-6}m
(ii) 3 km = 3×10^{3} m
(iii) 20 mm = 20×10^{-3}m
(iv) 73 pm = 73×10^{-12} m
(v) 7.5 nm = 7.5×10^{-9} m

Example 13. F = 5 N convert it into CGS system.

Solution : $F = 5\,\frac{kg \times m}{s^2} = (5)\,\frac{(10^3\,g)(100\ cm)}{s^2} = 5 \times 10^{5}\,\frac{g\ cm}{s^2}$ (in CGS system).

This unit ($\frac{g\ cm}{s^2}$) is also called dyne

Example 14. $G = 6.67 \times 10^{-11}\,\frac{m^3}{kg\ s^2}$ convert it into CGS system.

Solution : $G = 6.67 \times 10^{-11}\,\frac{m^3}{kg\ s^2} = (6.67{\times}10^{-11})\,\frac{(100\ cm)^3}{(1000g)s^2} = 6.67 \times 10^{-8}\,\frac{cm^3}{gs^2}$

Example 15. ρ = 2 g/cm^3 convert it into MKS system.

Solution : $\rho = 2\ g/cm^3 = (2)\,\frac{10^{-3}\ kg}{(10^{-2}m)^3}$

$= 2 \times 10^3\ kg/m^3$

Example 16. V = 90 km/hour convert it into m/s.

Solution : $V = 90\ km/hour = (90)\,\frac{(1000\ m)}{(60 \times 60\ second)}$

$V = (90)\left(\frac{1000}{3600}\right)\frac{m}{s}$

$V = 90 \times \frac{5}{18}\,\frac{m}{s}$

V = 25 m/s

7. POINT TO REMEMBER :

To convert km/hour into m/sec, multiply by $\frac{5}{18}$.

Solved Example

Example 17. Convert 7 pm into μm.

Solution : Let 7 pm = (x) μm, Now lets convert both LHS & RHS into meter
$7 \times (10^{-12})m = (x) \times 10^{-6}\ m$
get $x = 7 \times 10^{-6}$
So 7 pm = $(7{\times}10^{-6})$ μm

Some SI units of derived quantities are named after the scientist, who has contributed in that field a lot.

8. SI Derived units, named after the scientist :

S.N	Physical Quantity	SI Units			
		Unit name	**Symbol of the unit**	**Expression in terms of other units**	**Expression in terms of base units**
1.	Frequency $(f = \frac{1}{T})$	hertz	Hz	$\frac{\text{Oscillation}}{\text{s}}$	s^{-1}
2.	Force (F = ma)	newton	N	- - - - -	Kg m / s^2
3.	Energy, Work, Heat (W = Fs)	joule	J	Nm	Kg m^2 / s^2
4.	Pressure, stress $(P = \frac{F}{A})$	pascal	Pa	N / m^2	Kg / m s^2
5.	Power, $(\text{Power} = \frac{W}{t})$	watt	W	J / s	Kg m^2 / s^3
6.	Electric charge (q = it)	coulomb	C	- - - - - -	A s
7.	Electric Potential Emf. $(V = \frac{U}{q})$	volt	V	J / C	Kg m^2 / s^3 A
8.	Capacitance $(C = \frac{q}{v})$	farad	F	C / V	A^2 s^4 / kgm^2
9.	Electrical Resistance (V = i R)	ohm	Ω	V / A	kg m^2 / s^3 A^2
10.	Electrical Conductance $(C = \frac{1}{R} = \frac{i}{V})$	siemens (mho)	S , ℧	A / V	s^3 A^2 / kg m^2
11.	Magnetic field	tesla	T	Wb / m^2	Kg / s^2 A^1
12.	Magnetic flux	weber	Wb	V s or J/A	kg m^2 / s^2 A^1
13.	Inductance	henry	H	Wb / A	kg m^2 / s^2 A^2
14.	Activity of radioactive material	becquerel	Bq	$\frac{\text{Disintegration}}{\text{second}}$	s^{-1}

9. Some SI units expressed in terms of the special names and also in terms of base units:

Physical Quantity	SI Units	
	In terms of special names	**In terms of base units**
Torque (τ = Fr)	N m	Kg m^2 / s^2
Dynamic Viscosity (F_v = η A $\frac{dv}{dr}$)	Poiseiulle (P ℓ) or Pa s	Kg / m s
Impulse (J = F Δt)	N s	Kg m / s
Modulus of elasticity (Y = $\frac{\text{stress}}{\text{strain}}$)	N / m^2	Kg / m s^2
Surface Tension Constant (T) (T = $\frac{F}{\ell}$)	N/m or J/m^2	Kg / s^2
Specific Heat capacity (s) (Q = msΔT)	J/kg K (old unit s $\frac{\text{cal}}{\text{g. }^{o}\text{C}}$)	m^2 s^{-2} K^{-1}
Thermal conductivity (K) ($\frac{dQ}{dt}$ = KA $\frac{dT}{dr}$)	W / m K	m kg s^{-3} K^{-1}
Electric field Intensity E = $\frac{F}{q}$	V/m or N/C	m kg s^{-3} A^{-1}
Gas constant (R) (PV = nRT) or molar Heat Capacity (C = $\frac{Q}{M\Delta T}$)	J / K mol	m^2 kg s^{-2} K^{-1} mol^{-1}

10. CHANGE OF NUMERICAL VALUE WITH THE CHANGE OF UNIT :

Suppose we have

$$\ell = 7\text{ cm} \xrightarrow[\text{it into metres, we get}]{\text{If we convert}} = \frac{7}{100}\text{m}$$

we can say that if the unit is increased to 100 times (cm→m),

the numerical value became $\frac{1}{100}$ times $\left(7 \rightarrow \frac{7}{100}\right)$

So we can say

Numerical value $\propto \frac{1}{\text{unit}}$

We can also tell it in a formal way like the following :

Magnitude of a physical quantity = (Its Numerical value) (unit) = (n) (u)

Magnitude of a physical quantity always remains constant, it will not change if we express it in some other unit. So

$$\begin{tikzcd} & (n)\ (u) = \text{constant} \arrow[dl] \arrow[dr] & \\ n \propto \frac{1}{u} & & n_1u_1 = n_2u_2 \end{tikzcd}$$

numerical value $\propto \frac{1}{\text{unit}}$

Solved Example

Example 18. If unit of length is doubled, the numerical value of Area will be ....

Solution : As unit of length is doubled, unit of Area will become four times. So the numerical value of Area will became one fourth. Because numerical value $\propto \frac{1}{\text{unit}}$,

Example 19. Force acting on a particle is 5N.If unit of length and time are doubled and unit of mass is halved than the numerical value of the force in the new unit will be.

Solution : Force = 5 $\frac{\text{kg} \times \text{m}}{\text{sec}^2}$

If unit of length and time are doubled and the unit of mass is halved.

Then the unit of force will be $\left(\frac{\frac{1}{2} \times 2}{(2)^2}\right) = \frac{1}{4}$ times

Hence the numerical value of the force will be 4 times. (as numerical value $\propto \frac{1}{\text{unit}}$)

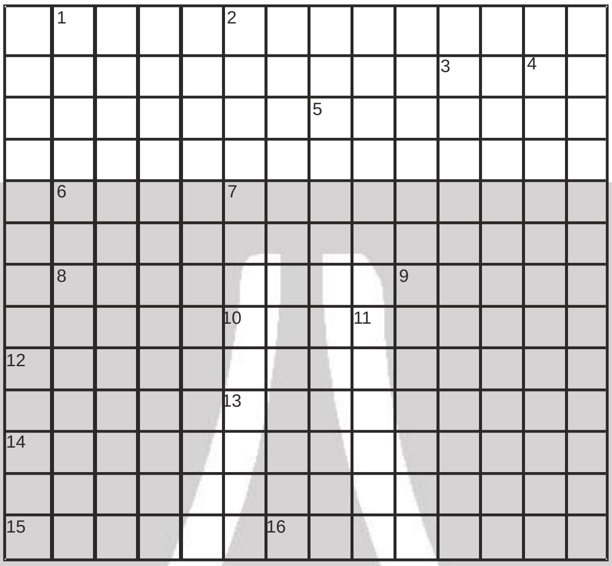
Note : ** Problems require knowledge of quantities from the syllabus of class XII.

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

****1.** Complete the cross word :



Across

1. Unit of pressure $\frac{N}{m^2}$ = ...(6)
3. Unit of a physical quantity whose dimension is $M'L^2T^{-3}$ ....(4)
6. Unit of conductance $\left(=\frac{1}{\text{Resistance}}\right)$ which is equivalent to Siemens ......(3)
7. A quantity whose dimension is same as that of energy. ....(6)
8. A unit of pressure (1mm of Hg pressure) ....(4)
10. Abbreviation used for 10^{-6} ....(5)
12. Nuclear distances are measured in .....(5)
13. Unit of luminous intensity ....(7)
14. Angular speed of a fan is usually written in ....(3)
15. erg/cm = ...(4)
16. Unit of inductance ....(5)

Down

1. 10^{-12} m = One ....(9)
2. A unit of length measures atomic distances ....(8)
3. Unit of magnetic flux ....(5)
4. Unit of magnetic field ....(5)
5. A unit of distance that is equal to 3.08×10^{16}m, and is used to measure astro nominal distances ....(6)
9. Number of particles is expressed in.......(4)
11. Unit of a physical quantity which is dimensionless ......(6)
12 Unit of capacitance ....(5)

2.✍ If the velocity of light 'c', Gravitational constant 'G' & Plank's constant 'h' be chosen as fundamental units, find the dimensions of mass, length & time in this new system .

3. Test if the following equations are dimensionally correct :

(a) $s = \rho gh / \cos\theta$ (b) $v = \sqrt{\frac{\gamma RT}{M_0}}$ (c) $V = \frac{Pr^4 t}{\eta \ell}$ (d) $f = \sqrt{\frac{mg\ell}{I}}$

where h = height, S = surface tension, v = Speed of sound, ρ = density, P = pressure, V = volume, η = coefficient of viscosity, f = frequency and I = moment of inertia.

PART - II : ONLY ONE OPTION CORRECT TYPE

1. Which of the following sets can’t enter into the list of fundamental quantities in any system of units?

(A) length, mass and velocity (B) length, time and velocity

(C) mass, time and velocity (D) length, time and mass

2.✍ A dimensionless quantity

(A) never has a unit (B) always has a unit (C) may have a unit (D) does not exit

3. A unit less quantity

(A) never has a nonzero dimension (B) always has a nonzero dimension

(C) may have a nonzero dimension (D) does not exit

4. Which pair of following quantities has dimensions different from each other.

(A) Impulse and linear momentum (B) Plank’s constant and angular momentum

(C) Moment of inertia and moment of force (D) Young’s modulus and pressure

5.✍ The velocity of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. The method of dimensions gives the relation between these quantities as

(A) $v^2 = k\lambda^{-1} g^{-1} \rho^{-1}$ (B) $v^2 = k g \lambda$ (C) $v^2 = k g \lambda \rho$ (D) $v^2 = k\lambda^3 g^{-1}\rho^{-1}$

where k is a dimensionless constant

6.✍ The value of $G = 6.67 \times 10^{-11}$ N m^2 (kg)$^{-2}$. Its numerical value in CGS system will be :

(A) 6.67×10^{-8} (B) 6.67×10^{-6} (C) 6.67 (D) 6.67×10^{-5}

7. Force applied by water stream depends on density of water (ρ), velocity of the stream (v) and cross-sectional area of the stream (A). The expression of the force can be

(A) ρAv (B) ρAv^2 (C) $\rho^2 Av$ (D) $\rho A^2 v$

8.✍ If unit of length and time is doubled, the numerical value of 'g' (acceleration due to gravity) will be :

(A) doubled (B) halved (C) four times (D) remain same

PART - III : MATCH THE COLUMN

1. **Match the following :**

	Physical quantity	**Dimension**	**Unit**
(1)	Gravitational constant 'G'	(P) $M^1L^1T^{-1}$	(a) N.m
(2)	Torque	(Q) $M^{-1}L^3T^{-2}$	(b) N.s
(3)	Momentum	(R) $M^1 L^{-1}T^{-2}$	(c) Nm^2/kg^2
(4)	Pressure	(S) $M^1L^2T^{-2}$	(d) pascal

2.** **Match the following :**

	Physical quantity	**Dimension**	**Unit**
(1)	Stefan's constant 'σ'	(P) $M^1L^1T^{-2}A^{-2}$	(a) W/m^2
(2)	Wien's constant 'b'	(Q) $M^1L^0T^{-3}K^{-4}$	(b) K.m.
(3)	Coefficient of viscosity 'η'	(R) $M^1L^0T^{-3}$	(c) tesla .m/A
(4)	Emissive power of radiation (Intensity emitted)	(S) $M^0L^1T^0K^1$	(d) $W/m^2.K^4$
(5)	Mutual inductance 'M'	(T) $M^1L^2T^{-2}A^{-2}$	(e) poise
(6)	Magnetic permeability 'μ_0'	(U) $M^1L^{-1}T^{-1}$	(f) henry

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1.✍ Force F is given in terms of time t and distance x by F = A sin C t + B cos Dx. Then the dimensions of $\frac{A}{B}$ and $\frac{C}{D}$ are given by

(A) $MLT^{-2}, M^0L^0T^{-1}$ (B) $MLT^{-2}, M^0L^{-1}T^0$ (C) $M^0L^0T^0, M^0L^1T^{-1}$ (D) $M^0L^1T^{-1}, M^0L^0T^0$

2.**✍ What are the dimensions of electrical resistance?

(A) $ML^2T^{-2}A^2$ (B) $ML^2T^{-3}A^{-2}$ (C) $ML^2T^{-3}A^2$ (D) $ML^2T^{-2}A^{-2}$

3. $\int \frac{x\,dx}{\sqrt{2ax-x^2}} = a^n \sin^{-1}\left[\frac{x}{a}-1\right]$. The value of n is :

(A) 0 (B) –1 (C) 1 (D) none of these

You may use dimensional analysis to solve the problem.

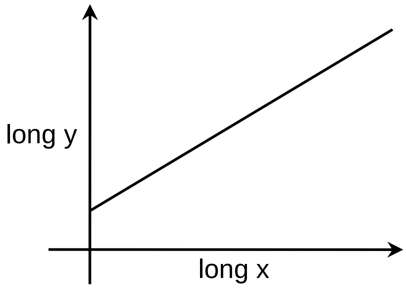
4.✍ An unknown quantity "α" is expressed as $\alpha = \frac{2ma}{\beta}\log\left(1+\frac{2\beta\ell}{ma}\right)$ where m = mass, a = acceleration, ℓ = length. The unit of α should be

(A) meter (B) m/s (C) m/s^2 (D) s^{-1}

5._ A quantity α is defined as $\alpha = \frac{e^2}{4\pi\varepsilon_0 c\hbar}$, where e is electric charge, $\hbar = \frac{h}{2\pi}$ is the reduced Planck's constant and c is the speed of light. The dimensions of α are **[Olympiad (State-1) 2017]**

(A) $[M^0 L^0 T^0 I^0]$ (B) $[M^1 L^{-1} T^2 I^{-2}]$ (C) $[M^2 L^1 T^{-1} I^0]$ (D) $[M^0 L^3 T^{-1} I^{-2}]$

6._ The equation correctly represented by the following graph is (a and b are constants)



[Olympiad (State-1) 2017]

(A) x + y = b (B) $ax^2 + by^2 = 0$ (C) x + y = ab (D) $y = ax^b$

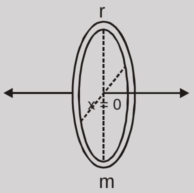
7._ The physical quantity that has unit volt-second is **[Olympiad (State-1) 2017]**

(A) energy (B) electric flux (C) magnetic flux (D) inductance

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1.✍ In the formula; $p = \frac{nRT}{V-b} e^{-\frac{a}{RTV}}$, find the dimensions of 'a' and 'b', where p = pressure, n = no. of moles, T = temperature, V = volume and R = universal gas constant.

2.✍ A particle is performing SHM along the axis of a fixed ring. Due to gravitational force, its displacement at time t is given by x = a sin ωt.



In this equation ω is found to depend on radius of the ring (r), mass of the ring (m) and gravitational constant (G). Using dimensional analysis, find the expression of ω in terms of m, r and G.

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1.✍ Choose the correct statement(s):

(A) All quantities may be represented dimensionally in terms of the base quantities.

(B) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.

(C) The dimension of a base quantity in other base quantities is always zero.

(D) The dimension of a derived quantity is never zero in any base quantity.

2.✍ Choose the correct statement(s) :

(A) A dimensionally correct equation may be correct.

(B) A dimensionally correct equation may be incorrect.

(C) A dimensionally incorrect equation may be correct.

(D) A dimensionally incorrect equation must be incorrect.

3. A parameter α is given by $\alpha = \frac{h}{\sigma\theta^4}$

(here σ = Stefan's constant, h = Planck's constant, θ = absolute temperature) then

(A) Dimension of 'α' will be $L^2\ T^2$

(B) Unit of 'α' may be $m^2\ s^2$

(C) Unit of 'α' may be $\frac{(\text{Weber})(\Omega)^2(\text{Farad})^2}{(\text{Tesla})}$

(D) Dimension of 'α' will be equal to dimension of $\left(\frac{R\,i}{\phi_m}\right)$ where R = gas constant, i = Electrical current, ϕ_m = magnetic flux

PART - IV : COMPREHENSION

Comprehension

The Vander waal equation for 1 mole of a real gas is $\left(P + \frac{a}{V^2}\right)$ (V – b) = RT where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are Vander waal constants.

1. The dimensions of a are the same as those of

(A) PV (B) PV^2 (C) P^2V (D) P/V

2. The dimensions of b are the same as those of

(A) P (B) V (C) PV (D) nRT

3. The dimensional formula for ab is

(A) $ML^2\ T^{-2}$ (B) $ML^4\ T^{-2}$ (C) $ML^6\ T^{-2}$ (D) $ML^8\ T^{-2}$

Exercise-3

✍ **Marked Questions can be used as Revision Questions.**

*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1.** Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II**. Match the physical quantities in **Column I** with the units in **Column II**. **[IIT-JEE-2007; 6/184]**

Column I	**Column II**
(A) GM_eM_s G - universal gravitational constant, M_e - mass of the earth, Ms - mass of the Sun	(p) (volt) (coulomb) (metre)
(B) $\frac{3RT}{M}$ R - universal gas constant, T - absolute temperature, M - molar mass	(q) (kilogram) $(\text{metre})^3$ $(\text{second})^{-2}$
(C) $\frac{F^2}{q^2B^2}$ F - force, q - charge, B - magnetic field	(r) $(\text{metre})^2$ $(\text{second})^{-2}$
(D) $\frac{GM_e}{R_e}$ G - universal gravitational constant, M_e - mass of the earth R_e - radius of the earth	(s) (farad) $(\text{volt})^2$ $(\text{kg})^{-1}$

2.✍ Match List I with List II and select the correct answer using the codes given below the lists :

[JEE (Advanced) 2013; 4/60]

List I	**List II**
P. Boltzmann constant	1. $[ML^2T^{-1}]$
Q. Coefficient of viscosity	2. $[ML^{-1}T^{-1}]$
R. Planck constant	3. $[MLT^{-3}K^{-1}]$
S. Thermal conductivity	4. $[ML^2T^{-2}K^{-1}]$

Codes :

	P	Q	R	S
(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

3. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer find that d is proportional to $S^{1/n}$. The value of n is: **[JEE (Advanced) 2014, P-1, 3/60]**

4.* Planck's constant h, speed of light c and gravitational constant G are used to from a unit of length L and a unit of mass M. Then the correct options(s) is (are) **[JEE (Advanced) 2015 ; P-1, 4/88, –2]**

(A) $M \propto \sqrt{c}$ (B) $M \propto \sqrt{G}$ (C) $L \propto \sqrt{h}$ (D) $L \propto \sqrt{G}$

5.* In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equations(s) is(are) **[JEE (Advanced) 2015 ; P-2, 4/88, –2]**

(A) $\mu_0 I^2 = \varepsilon_0 V^2$ (B) $\varepsilon_0 I = \mu_0 V$ (C) $I = \varepsilon_0 V$ (D) $\mu_0 c I = \varepsilon_0 V$

6. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to **[JEE (Advanced) 2017; P-2, 3/61, –1]**

(A) R^3 (B) R (C) $R^{2/3}$ (D) 1/R

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while [ε_0] and [μ_0] stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on PARAGRAPH "X", the question given below is one of them)

[JEE (Advanced) 2018; P-1, 3/60, –1]

7. The relation between [E] and [B] is

(A) [E] = [B] [L] [T] (B) [E] = [B] $[L]^{-1}$ [T] (C) [E] = [B] [L] $[T]^{-1}$ (D) [E] = [B] $[L]^{-1}$ $[T]^{-1}$

8. The relation between [ε_0] and [μ_0] is

(A) $[\mu_0] = [\varepsilon_0] [L]^2 [T]^{-2}$ (B) $[\mu_0] = [\varepsilon_0] [L]^{-2} [T]^2$ (C) $[\mu_0] = [\varepsilon_0]^{-1} [L]^2 [T]^{-2}$ (D) $[\mu_0] = [\varepsilon_0]^{-1} [L]^{-2} [T]^2$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Which of the following units denotes the dimensions ML^2/Q^2, where Q denotes the electric charge?

[AIEEE-2006, 3/180]

(1) H/m^2 (2) Weber (Wb) (3) Wb/m^2 (4) Henry (H)

2.✍ The dimension of magnetic field in M, L, T and C (Coulomb) is given as **[AIEEE-2008, 3/105]**

(1) MT^2C^{-2} (2) $MT^{-1}C^{-1}$ (3) $MT^{-2}C^{-1}$ (4) $MLT^{-1}C^{-1}$

3.✍ Let [$\in_0$] denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then : **[JEE(Main) 2013, 4/120, –1]**

(1) $[\in_0] = [M^{-1}L^{-3}T^2A]$ (2) $[\in_0] = [M^{-1}L^{-3}T^4A^2]$ (3) $[\in_0] = [M^{-1}L^2T^{-1}A^{-2}]$ (4) $[\in_0] = [M^{-1}L^2T^{-1}A]$

4. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by factor of : **[JEE (Main) 2017, 4/120, –1]**

(1) $\frac{1}{81}$ (2) 9 (3) $\frac{1}{9}$ (4) 81

Answers

EXERCISE-1

PART - I

1.

	^{1}P	A	S	C	^{2}A	L							
	I				N					^{3}W	A	^{4}T	T
	C				G		^{5}P			E		E	
	O				S		A			B		S	
	^{6}M	H	O		^{7}T	O	R	Q	U	E		L	
	E				R		S			R		A	
	^{8}T	O	R	R	O		E		^{9}M				
	R				^{10}M	I	C	^{11}R	O				
^{12}F	E	R	M	I				A	L				
A					^{13}C	A	N	D	E	L	A		
^{14}R	P	M						I					
A								A					
^{15}D	Y	N	E			^{16}H	E	N	R	Y			

2. $[M] = [h^{1/2}.C^{1/2}.G^{-1/2}]$; $[L] = [h^{1/2}. C^{-3/2}.G^{1/2}]$;
$[T] = [h^{1/2} . C^{-5/2} . G^{1/2}]$

3. All are dimensionally correct.

PART - II

1.	(B)	**2.**	(C)	**3.**	(A)
4.	(C)	**5.**	(B)	**6.**	(A)
7.	(B)	**8.**	(A)		

PART - III

1. (1) → (Q) → (c) ; (2) → (S) → (a)
(3) → (P) → (b) ; (4) → (R) → (d)

2. (1) → (Q) → (d) ; (2) → (S) → (b)
(3) → (U) → (e) ; (4) → (R) → (a)
(5) → (T) → (f) ; (6) → (P) → (c)

EXERCISE-2

PART - I

1.	(C)	**2.**	(B)	**3.**	(C)
4.	(A)	**5.**	(A)	**6.**	(D)
7.	(C)				

PART - II

1. $[a] = ML^5T^{-2}mol^{-1}[b] = L^3$

2. ω = (some number) $\sqrt{\frac{Gm}{r^3}}$.

PART - III

1.	(ABC)	**2.**	(ABD)	**3.**	(ABC)

PART - IV

1.	(B)	**2.**	(B)	**3.**	(D)

EXERCISE-3

PART – I

1. (A) → (p), (q) ; (B) → (r), (s) ;
(C) → (r), (s) ; (D) → (r), (s)

2.	(C)	**3.**	3	**4.**	(ACD)
5.	(AC)	**6.**	(B)	**7.**	(C)
8.	(D)				

PART - II

1.	(4)	**2.**	(2)	**3.**	(2)
4.	(2)				

HINT OF SOLUTION OF UNIT & DINMENSION

EXERCISE-1

PART - I

भाग - I

1.

	^{1}P	A	S	C	^{2}A	L							
	I				N					^{3}W	A	^{4}T	T
	C				G		^{5}P			E		E	
	O				S		A			B		S	
	^{6}M	H	O		^{7}T	O	R	Q	U	E		L	
	E				R		S			R		A	
	^{8}T	O	R	R	O		E		^{9}M				
	R				^{10}M	I	C	^{11}R	O				
^{12}F	E	R	M	I				A	L				
A					^{13}C	A	N	D	E	L	A		
^{14}R	P	M						I					
A								A					
^{15}D	Y	N	E			^{16}H	E	N	R	Y			

Answer is itself the solution. उत्तर ही स्वयं के लिए हल है।

2. We have the equation हमारे पास समीकरण है

$$\frac{Gm_1m_2}{r^2} = F$$

$$\frac{[G][M]^2}{[L]^2} = MLT^{-2}$$

$[G] = M^{-1}L^3T^{-2}$ .... (i)

$$\frac{hc}{\lambda} = \text{Energy} \text{ ऊर्जा}$$

$$\frac{[h]\ \ [c]}{[\lambda]} = ML^2T^{-2} \quad [c] = LT^{-1}$$

$[\lambda] = L$

$[h] = ML^2T^{-1}$ .... (ii)

$[c] = LT^{-1}$ .... (iii)

taking the product of (i) & (ii) समीकरण (i) व (ii) का गुणन करने पर

$[G]\,[h] = L^5T^{-3}$

$[c]^3 = L^3T^{-3}$

$$\therefore \quad \frac{[G][h]}{[c]^3} = L^2$$

$$G^{1/2}h^{1/2}c^{-3/2} = L$$

again from (iii) दुबारा समीकरण (iii) से

$$[T] = \frac{[L]}{[c]} = G^{1/2}h^{1/2}c^{-3/2-1} = G^{1/2}h^{1/2}c^{-5/2}$$

From (ii) समीकरण (ii) से

$[h] = ML^2T^{-1}$

$$[h] = \frac{MGhc^{-3}}{G^{1/2}h^{1/2}c^{-5/2}}$$

$[h] = MG^{1/2}h^{1/2}c^{-3+5/2}$

or $G^{-1/2}\ h^{1/2}\ c^{1/2} = M$

3. All are dimensionally correct. विमीय रूप से सभी सत्य है।

PART - II

भाग - II

1. Velocity depends on length and time, so cannot be taken as base quantities.
वेग, दूरी और समय पर निर्भर करता है अतः इसे मूल राशि नही मान सकते है।

2. Angle is dimensionless but has unit (radian or degree)
कोण विमाहीन है लेकिन मात्रक रेडियन या डिग्री हो सकता है।

3. It is obvious. यह स्वतः स्पष्ट है।

4. [moment of force] = [F] [d] = ML^2T^{-2} .
[Moment of Inertia] = [I] = ML^2
[बलाघूर्ण] = [F] [d] = ML^2T^{-2} .
[जड़त्व आघूर्ण] = [I] = ML^2

5. $[v] = [k]\ [\lambda^a\ \rho^b\ g^c] \quad \Rightarrow \quad LT^{-1} = L^a\ M^b\ L^{-3b}\ L^c\ T^{-2c}$
$\Rightarrow \quad LT^{-1} = M^b\ L^{a-3b+c}\ T^{-2c}$
$\Rightarrow \quad a = ½,\ b = 0,\ c = ½$
so, $v^2 = kg\lambda$

6. $G = 6.67 \times 10^{-11}\ N\ m^2\ (kg)^{-2}$
$= 6.67 \times 10^{-11} \times 10^5\ dyne \times 100^2\ cm^2 / (10^3)^2\ g^2 = 6.67 \times 10^{-8}\ dyne\text{-}cm^2\text{-}g^{-2}$

7. It is obvious
यह स्वयं स्पष्ट है।

8. $[g] = LT^{-2}$ and numerical value $\propto \dfrac{1}{unit}$

$[g] = LT^{-2}$ और गणितीय मान $\propto \dfrac{1}{\text{मात्रक}}$

PART - III

भाग - III

1. $F = G\frac{m_1m_2}{r^2} \Rightarrow [G] = \frac{[F][r^2]}{[m_1m_2]} = \frac{MLT^{-2}L^2}{M^2}$

$= M^{-1} L^3 T^{-2}$

[Torque] [बलाघूर्ण] = $[f]\ [d] = MLT^{-2}L = ML^2T^{-2}$

[Momentum] [संवेग]= $[m]\ [v] = MLT^{-1}$

$[p] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$.

2. (i) $U = \sigma AT^4 \Rightarrow [\sigma] = \frac{[U]}{[A][T^4]} = \frac{ML^2T^{-3}}{L^2K^4} = MT^{-3}\ K^{-4}$

(ii) $\lambda T = b \Rightarrow [b] = [\lambda]\ [T] = LK$

(iii) $F = 6\pi\eta rv \Rightarrow [\eta] = \frac{[F]}{[r][v]} = \frac{MLT^{-2}}{L.\ LT^{-1}} = ML^{-1}\ T^{-1}$

(iv) $I = \frac{P}{A} = \frac{ML^2T^{-3}}{L^2} = ML^o\ T^{-3}$

(v) Energy ऊर्जा $= \frac{1}{2} Mi^2 \Rightarrow [M] = \frac{[E]}{[i^2]} = ML^2T^{-2}\ A^{-2}$

(vi) $\frac{[U]}{[V]} = \frac{[B^2]}{[2\mu_0]}$

$= \quad [\mu_0] = \frac{[B^2]\ [V]}{[U]}$

Also , $F = qVB \Rightarrow B = \frac{F}{qv}$

$[\mu_0] = \frac{(F)^2[V]}{[q^2v^2][U]} = MLT^{-2}A^{-2}$ **Ans.**

EXERCISE-2

PART - I

1. All the terms in the equation must have the dimension of force

$\therefore\ [A \sin C\ t] = MLT^{-2}$

$\Rightarrow \quad [A]\ [M^0L^0T^0] = MLT^{-2}$

$\Rightarrow \quad [A] = MLT^{-2}$

Similarly, $[B] = MLT^{-2}$

$\therefore \quad \frac{[A]}{[B]} = M^0L^0T^0$

Again $[Ct] = M^0L^0T^0 \Rightarrow [C] = T^{-1}$

$[Dx] = M^0L^0T^0 \Rightarrow [D] = L^{-1}$

$\Rightarrow \quad \frac{[C]}{[D]} = M^0L^1T^{-1}$.

समीकरण में सभी पदों की विमा बल की विमा होनी चाहिए

$\therefore \quad [A \sin C t] = MLT^{-2}$

$\Rightarrow \quad [A]\,[M^0L^0T^0] = MLT^{-2}$

$\Rightarrow \quad [A] = MLT^{-2}$

इसी तरह, $[B] = MLT^{-2}$

$\therefore \quad \frac{[A]}{[B]} = M^0L^0T^0$

पुनः $\quad [Ct] = M^0L^0T^0 \quad \Rightarrow \quad [C] = T^{-1}$

$\quad [Dx] = M^0L^0T^0 \quad \Rightarrow \quad [D] = L^{-1}$

$\Rightarrow \quad \frac{[C]}{[D]} = M^0L^1T^{-1}$

2. V = IR

V has the dimensions of की विमा है

$$[V] = \frac{[\text{work}]}{[\text{charge}]} = \frac{ML^2T^{-2}}{AT} = ML^2\,T^{-3}\,A^{-1}$$

$$\therefore \; [R] = \frac{[v]}{[I]} = ML^2\,T^{-3}\,A^{-2}$$

V = IR

V has the dimensions of की विमा है

$$[V] = \frac{[\text{work}]}{[\text{charge}]} = \frac{ML^2T^{-2}}{AT} = ML^2\,T^{-3}\,A^{-1}$$

$$\therefore \; [R] = \frac{[v]}{[I]} = ML^2\,T^{-3}\,A^{-2}$$

3. $$\int \frac{x\,dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x}{a} - 1\right]$$

denominator $2ax - x^2$ must have the dimension of $[x]^2$

($\because$ we can add or substract only if quantities have same dimension)

हर $2ax - x^2$ की विमायें $[x]^2$ ही होनी चाहिये

($\because$ हम केवल तब ही जोड या घटा सकते हैं, जब राशियों की विमायें समान है।)

$$\therefore \left[\sqrt{2ax - x^2}\right] = [x]$$

Also, dx has the dimension of [x]   dx की विमा भी वही है जो [x] की है।

$\therefore \frac{x \;\; dx}{\sqrt{2ax - x^2}}$ is having dimension L

$\therefore \frac{x \;\; dx}{\sqrt{2ax - x^2}}$ की विमा L है

Equating the dimension of L.H.S. & R.H.S. we have

दायें पथ व बायें पथ की विमायें समान करने पर

$[a^n] = M^0L^1T^0$ $\{\therefore \sin^{-1}\left(\frac{x}{a} - 1\right)$ must be dimensionless विमाहीन होना चाहिये $\}$

$\therefore n = 1$

4. $[\alpha] = \left[\frac{ma}{\beta}\right]$ ....(i)

$\left[\frac{\beta}{ma}\right][\ell] = M^0L^0T^0$

$\Rightarrow \quad \left[\frac{ma}{\beta}\right] = [\alpha] = [\ell] = L$

5. $[\alpha] = \left[\frac{e^2}{\varepsilon_0}\right]\left[\frac{1}{hc}\right]$

$= [Fr^2]\ \frac{1}{[E\lambda]}$

$= [M^1L^1T^{-2}L^2]\frac{1}{[M^1L^2T^{-2}L^1]} = [M^1L^3T^{-2}\ \ M^{-1}L^{-3}T^2] = [M^0\ L^0\ T^0]$

6. $\log y = m \log x + C$

$\log y = \log c'x^m$

$y = c'x^m$

$y = ax^b$

7. $Li = \frac{Li^2}{i} = \frac{Vq}{i}$ = volt – second

PART - II

1. $[b] = [V] = L^3$

$[a] = [RTV] = \frac{[PV]}{[n]}\ .\ [V] = \frac{ML^2T^{-2}L^3}{mol}$

$= ML^5\ T^{-2}\ mol^{-1}$.

2. Let, $\omega = KM^ar^bG^c$ where K is a dimensionless constant.
Writing the dimension of both the sides and equating then we have,

$T^{-1} = M^aL^b(M^{-1}L^3T^{-2})^c$

$= M^{a-c}\,L^{b+3c}\,T^{-2c}$

Equating the exponents

$-2c = -1$ or $c = \frac{1}{2}$,

$b + 3c = 0$ or $-3c = b = -3/2$

$a - c = 0\ .\ c = a = +1/2$

Thus the required equation is $\omega = K\sqrt{\frac{Gm}{r^3}}$

माना, $\omega = KM^ar^bG^c$ यहाँ K एक विमाहीन नियतांक है

दोनों ओर विमायें लिखते हुए बराबर करने पर

$T^{-1} = M^aL^b(M^{-1}L^3T^{-2})^c$

$= M^{a-c}\,L^{b+3c}\,T^{-2c}$

चरघातांकों को बराबर करने पर

$-2c = -1$ or $c = 1/2$,

$b + 3c = 0$ or $-3c = b = -3/2$

$a - c = 0\ .\ c = a = +1/2$

पूछी गयी समीकरण है $\omega = K\sqrt{\frac{Gm}{r^3}}$

PART - III

1. All A, B & C are obvious.
A, B व C सभी स्वतः स्पष्ट है।

2. It is obvious

3. $[\alpha] = \frac{[h]}{[\sigma\theta^4]} = \frac{ML^2T^{-1}}{MT^{-3}K^{-4}.K^4} = L^2\,T^2$

So, unit of α will be $m^2s^{2.}$

अतः, α का मात्रक m^2s^2 होगा।

$$\frac{(\text{weber})\ \ (\Omega)^2(\text{Farad})^2}{\text{Tesla}} = \frac{Tm^2.\ \ \Omega^2F^2}{T} = m^2s^2$$

PART - IV

1. $[P] = \left[\frac{a}{V^2}\right] \Rightarrow\ [a] = [P]\,[V^2]$

2. $[b] = [V]$

3. $[a]\,[b] = [PV^2]\,[V] = [P]\,[V^3] = ML^{-1}\,T^{-2}\,[L^3]^3\ = ML^8\,T^{-2}$

EXERCISE-3

PART - I

भाग - I

1. (A) $\frac{GM_eM_s}{R_e^{\,2}}$ = Force

$[GM_eM_s]$ = [Force] [बल] $[R_e^2]$

$= MLT^{-2}\,L^2\ = ML^3T^{-2}$

Hence SI unit of GM_eM_s, will be (kilogram) (meter^3)(sec^{-2})

ie same as (volt) (coulomb) (metre)

अतः GM_eM_s का SI मात्रक होगा (किग्रा) (मी^{3}) (सेकण्ड$^{-2}$)

जो कि समान है (वोल्ट) (कूलाम) (मीटर)

(B) $\sqrt{\frac{3RT}{M}} = V_{R.M.S.}$

$\left[\frac{3RT}{M_0}\right] = [V_{R.M.S.}]^2\ = L^2T^{-2}$

Hence SI unit will be (metre)^2 (second)$^{-2}$ ie same as (farad) (volt)^2 (kg)$^{-1}$

अतः SI मात्रक होगा (मी)^2 (सेकण्ड)$^{-2}$ जो कि समान है (फैरड) (वोल्ट)^2 (kg)$^{-1}$

(C) $\frac{[F^2]}{[q^2B^2]} = \frac{[q^2v^2B^2]}{[q^2B^2]} = [V^2] = L^2T^{-2}$

Hence SI unit (metre)^2 (second)$^{-2}$ i.e. same as (farad) (volt)^2 (kg)$^{-1}$

अतः SI मात्रक (मी)^2 (सेकण्ड)$^{-2}$ जो कि समान है (फैरड) (वोल्ट)^2 (kg)$^{-1}$

(D) $\left[\frac{GM_e}{R_e}\right] = \frac{[\text{Force}]\ \ [R_e]}{[\text{Mass}]} = \frac{MLT^{-2}L}{M} = L^2T^{-2}$

Hence SI unit will be (meter)$^{-2}$ (second)$^{-2}$ i.e. same as (farad) (volt)^2 (kg)$^{-1}$

अतः SI मात्रक (मी)^2 (सेकण्ड)$^{-2}$ जो कि समान है (फैरड) (वोल्ट)^2 (kg)$^{-1}$

2. (p) $U = \frac{1}{2}kT \Rightarrow ML^2T^{-2} = [k]\ K \Rightarrow [K] = ML^2T^{-2}K^{-1}$

(q) $F = \eta A\frac{dv}{dx} \Rightarrow [\eta] = \frac{MLT^{-2}}{L^2LT^{-1}L^{-1}} = ML^{-1}\ T^{-1}$

(r) $E = h\nu \Rightarrow ML^2T^2 = [h]\ T^{-1} \Rightarrow [h] = ML^2\ T^{-1}$

(s) $\frac{dQ}{dt} = \frac{kA\Delta\theta}{\ell} \Rightarrow [k] = \frac{ML^2T^{-3}L}{L^2K} = MLT^{-3}\ K^{-1}$

3. $d = k\ (\rho)^a\ (S)^b\ (f)^c$

$\Rightarrow [L] = \left[\frac{M}{L^3}\right]^a \left[\frac{M^1L^2T^{-2}}{L^2\ \ T}\right]^b \left[\frac{1}{T}\right]^c$

$0 = a + b$

$1 = -3a \Rightarrow a = -\frac{1}{3}$ So अतः $b = \frac{1}{3}$

$0 = -3b + c$

So अतः n = 3

4.* $M = h^x\ c^y\ G^z$

$M = (ML^2T^{-1})^x\ (LT^{-1})^y\ (M^{-1}L^3T^{-2})^z$

$x - z = 1$

$2x + y + 3z = 0$

$-x - y - 2z = 0$

$x = \frac{1}{2}, \quad y = \frac{1}{2}, \quad z = \frac{-1}{2}$

$M\ \alpha\ \sqrt{h}\ \sqrt{c}\ \frac{1}{\sqrt{G}}$

For L

$x - z = 0$

$2x + y + 3z = 1$

$-x - y - 2z = 0$

$x = \frac{1}{2} \quad y = \frac{-3}{2} \quad z = \frac{1}{2}$

$L\ \alpha\ \sqrt{h}\ \frac{1}{C^{3/2}}\ \sqrt{G}$

5.* (A) Energy of inductor प्रेरकत्व की ऊर्जा **=** $\frac{1}{2}LI^2 = \frac{1}{2}\frac{M_0N^2A}{\ell}I^2$

Energy of capacitor संधारित्र की ऊर्जा **=** $\frac{1}{2}CV^2 = \frac{1}{2}\in_0\frac{A}{d}V^2$

$\mu_0\frac{A}{\ell}I^2$ & $\in_0\frac{A}{d}V^2$ have same dimension समान विमा है

So इसलिये $\mu_0 I^2$ & $\in_0 V^2$ have same dimension समान विमा है

(C) $Q = CV$

$\frac{Q}{t} = \frac{CV}{t}$

$I = \in_0\frac{A}{\ell}\frac{V}{t}$

$\frac{A}{\ell t}$ have unit of speed चाल का मात्रक है

So $I = \in_0 cV$

6. $m = \frac{4\pi R^3}{3} \times \rho$

$$\ell n(m) = \ell n\left(\frac{4\pi}{3}\right) + \ell n(\rho) + 3\ell n(R)$$

$$0 = 0 + \frac{1}{\rho}\frac{d\rho}{dt} + \frac{3}{R}\frac{dR}{dt}$$

$$\Rightarrow \left(\frac{dR}{dt}\right) = v \propto -R \times \frac{1}{\rho}\left(\frac{d\rho}{dt}\right)$$

$v \propto R$

7. In terms of dimension

विमा के पदों में

$qE = qvB$

$E = vB$

$[E] = [B]\,[LT^{-1}]$

8. $C = \frac{1}{\sqrt{\mu_0 \in_0}}$

$C^2 = \frac{1}{\mu_0 \in_0}$

$\mu_0 = \in_0 . C^2$

$[\mu_0] = [\in_0^{-1}]^{-1} L^{-2} T^2$

PART - II
भाग - II

1. Energy stored in inductor

प्रेरक मे संचित ऊर्जा

$U = \frac{1}{2}LI^2 \qquad \Rightarrow \qquad L = \frac{2U}{I^2}$

$[L] = \frac{ML^2T^{-2}}{Q^2/T^2} = \frac{ML^2}{Q^2}$

Since Henry is unit of inductance L

प्रेरक L का मात्रक हैनरी है।

$\therefore$ (4) is correct. सही है।

2. From $F = qvB$ से

$\Rightarrow$ $[MLT^{-2}] = [C]\,[LT^{-1}]\,[B]$

$\Rightarrow$ $[B] = [MC^{-1}T^{-1}]$

3. $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{R^2}$

$\varepsilon_0 = \frac{q_1q_2}{4\pi FR^2}$

Hence $\varepsilon_0 = \frac{C^2}{N.m^2} = \frac{[AT]^2}{MLT^{-2}\,.\,L^2} = [M^{-1}\,L^{-3}\,T^4A^2]$

Ans. (2)

4. volume of man becomes = $(9)^3$ times

weight of man becomes = 9^3 times

Cross section area in leg = 9^2 times

stress = $\frac{\text{weight}}{\text{Area}}$ = 9 times

आदमी का आयतन = $(9)^3$ गुना हो जायेगा

आदमी का भार = 9^3 गुना हो जायेगा

पैर का अनुप्रस्थ काट क्षेत्र = 9^2 गुना हो जायेगा

प्रतिबल = $\frac{\text{भार}}{\text{क्षेत्रफल}}$ = 9 गुना हो जायेगा

HANDOUT **UNIT & DIMENSION**

PHYSICAL QUANTITIES :

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities. Till class X we have studied many physical quantities eg. length, velocity, acceleration, force, time, pressure, mass, density etc.

Physical quantities are of three types

- Fundamental or Basic quantities
- Derived Quantities
- Supplementary Quantities

1. Fundamental (Basic) Quantities :

- These are the elementary quantities which covers the entire span of physics.
- Any other quantities can be derived from these.
- All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance (d), time (t) and velocity (v) cannot be chosen as basic quantities (because they are related as V = d/t). An International Organization named CGPM : General Conference on weight and Measures, chose seven physical quantities as basic or fundamental.

- Length (L)
- Time (T)
- Mass (M)
- Temperature (K)
- Electrical current (A)
- Luminous Intensity (Cd)
- Amount of Substance (mol)

These are the elementary quantities (in our planet) that's why chosen as basic quantities.

In fact any set of independent quantities can be chosen as basic quantities by which all other physical quantities can be derived.

i.e.,

- (A) Area
- (V) Velocity
- (ρ) Density
- (S) Sp.Heat Capacity
- (R) Resistance
- (Cd) Luminous Intensity
- (mol) mole

Can be chosen as basic quantities (on some other planet, these might also be used as basic quantities)

But

- (L) Length
- (A) Area
- (V) Velocity

cannot be used as basic quantities as

Area = (Length)^2 so they are not independent.

2. Derived Quantities :

Physical quantities which can be expressed in terms of basic quantities (M,L,T....) are called derived quantities.

i.e., Momentum $P = mv = (m)\dfrac{\text{displacement}}{\text{time}} = \dfrac{ML}{T} = M^1 L^1 T^{-1}$

Here [$M^1 L^1 T^{-1}$] is called dimensional formula of momentum, and we can say that momentum has

1 Dimension in M (mass)

1 Dimension in L (length)

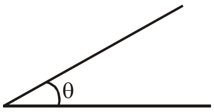
and –1 Dimension in T (time)

The representation of any quantity in terms of basic quantities (M, L, T....) is called dimensional formula and in the representation, the powers of the basic quantities are called dimensions.

3. Supplementary quantities :

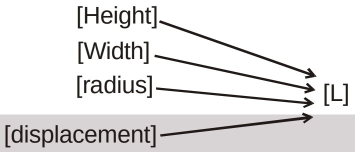
Besides seven fundamental quantities two supplementary quantities are also defined. They are

- Plane angle (The angle between two lines)
- Solid angle



FINDING DIMENSIONS OF VARIOUS PHYSICAL QUANTITIES :

- Height, width, radius, displacement etc. are a kind of length. So we can say that their dimension is [L]



here [Height] can be read as "Dimension of Height"

- Area = Length × Width

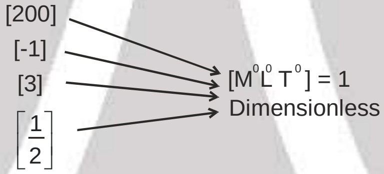
So, dimension of area is [Area] = [Length] × [Width] = [L] × [L] = $[L^2]$

For circle

Area = πr^2

$[\text{Area}] = [\pi]\,[r^2] = [1]\,[L^2] = [L^2]$

Here π is not a kind of length or mass or time so π shouldn't affect the dimension of Area. Hence its dimension should be 1 ($M^0L^0T^0$) and we can say that it is dimensionless. From similar logic we can say that all the numbers are dimensionless.



- [Volume] = [Length] × [Width] × [Height]

$= L \times L \times L = [L^3]$

For sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$[\text{Volume}] = \left[\frac{4}{3}\pi\right][r^3] = (1)\,[L^3] = [L^3]$$

So dimension of volume will be always $[L^3]$ whether it is volume of a cuboid or volume of sphere.

<u>Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.</u>

- $$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$[\text{Density}] = \frac{[\text{mass}]}{[\text{volume}]} = \frac{M}{L^3} = [M^1L^{-3}]$$

- $$\text{Velocity (v)} = \frac{\text{displacement}}{\text{time}}$$

$$[v] = \frac{[\text{Displacement}]}{[\text{time}]} = \frac{L}{T} = [M^0L^1T^{-1}]$$

- Acceleration (a) = dv/dt

$$[a] = \frac{dv}{dt}\ \frac{\text{kind of velocity}}{\text{kind of time}} = \frac{LT^{-1}}{T} = LT^{-2}$$

- Momentum (P) = mv

$[P] = [M]\,[v] = [M]\,[LT^{-1}] = [M^1L^1T^{-1}]$

- Force (F) = ma

$[F] = [m]\,[a] = [M]\,[LT^{-2}] = [M^1L^1T^{-2}]$

(You should remember the dimensions of force because it is used several times)

- Work or Energy = force × displacement

  [Work] = [force] [displacement]

  $= [M^1L^1T^{-2}]\ [L] = [M^1L^2T^{-2}]$

- Power = $\frac{\text{work}}{\text{time}}$

  $[\text{Power}] = \frac{[\text{work}]}{[\text{time}]} = \frac{M^1L^2T^{-2}}{T} = [M^1L^2T^{-3}]$

- Pressure = $\frac{\text{Force}}{\text{Area}}$

  $[\text{Pressure}] = \frac{[\text{Force}]}{[\text{Area}]} = \frac{M^1L^1T^{-2}}{L^2} = M^1L^{-1}T^{-2}$

1. Dimensions of angular quantities :

- Angle (θ)

  (Angular displacement) $\theta = \frac{\text{Arc}}{\text{radius}}$

  $[\theta] = \frac{[\text{Arc}]}{[\text{radius}]} = \frac{L}{L} = [M^0L^0T^0]$ (Dimensionless)

- Angular velocity (ω) = $\frac{\theta}{t}$

  $[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = [M^0L^0T^{-1}]$

- Angular acceleration (α) = $\frac{d\omega}{dt}$

  $[\alpha] = \frac{[d\omega]}{[dt]} = \frac{M^0L^0T^{-1}}{T} = [M^0L^0T^{-2}]$

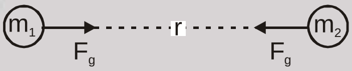
- Torque = Force × Arm length

  [Torque] = [force] × [arm length]

  $= [M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$

2. Dimensions of Physical Constants :

- **Gravitational Constant :**



If two bodies of mass m_1 and m_2 are placed at r distance, both feel gravitational attraction force, whose value is,

Gravitational force $F_g = \frac{Gm_1m_2}{r^2}$

where G is a constant called Gravitational constant.

$$[F_g] = \frac{[G][m_1][m_2]}{[r^2]}$$

$$[M^1L^1T^{-2}] = \frac{[G][M][M]}{[L^2]}$$

$$[G] = M^{-1}L^3T^{-2}$$

- **Specific heat capacity :**

  To increase the temperature of a body by ΔT, Heat required is Q = ms ΔT

  Here s is called specific heat capacity.

  $[Q] = [m]\ [s]\ [\Delta T]$

  Here Q is heat : A kind of energy so $[Q] = M^1L^2T^{-2}$

  $[M^1L^2T^{-2}] = [M]\ [s]\ [K]$

  $[s] = [M^0L^2T^{-2}K^{-1}]$

- **Gas constant (R) :**

  For an ideal gas, relation between pressure (P)
  Value (V), Temperature (T) and moles of gas (n) is
  PV = nRT where R is a constant, called gas constant.
  [P] [V] = [n] [R] [T] .... (1)

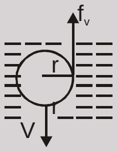
  here $[P]\,[V] = \frac{[\text{Force}]}{[\text{Area}]}\,[\text{Area} \times \text{Length}]$
  = [Force] × [Length]
  $= [M^1L^1T^{-2}]\,[L^1] = M^1L^2T^{-2}$
  From equation (1)
  [P] [V] = [n] [R] [T]
  $\Rightarrow [M^1L^2T^{-2}] = [\text{mol}]\,[R]\,[K]$
  $\Rightarrow [R] = [M^1L^2T^{-2}\,\text{mol}^{-1}\,K^{-1}]$

- **Coefficient of viscosity :**

  If any spherical ball of radius r moves with velocity v in a viscous liquid, then viscous force acting on it is given by

$F_v = 6\pi\eta rv$
Here η is coefficient of viscosity

$[F_v] = [6\pi]\,[\eta]\,[r]\,[v]$
$M^1L^1T^{-2} = (1)\,[\eta]\,[L]\,[LT^{-1}]$
$[\eta] = M^1L^{-1}T^{-1}$



- **Planck's constant :**

  If light of frequency υ is falling, energy of a photon is given by
  $E = h\upsilon$ Here h = Planck's constant
  $[E] = [h]\,[\upsilon]$

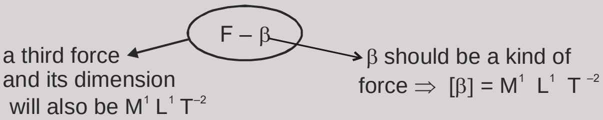
  $\upsilon = \text{frequency} = \frac{1}{\text{Time Period}} \Rightarrow [\upsilon] = \frac{1}{[\text{Time Period}]} = \left[\frac{1}{T}\right]$

  so $M^1L^2T^{-2} = [h]\,[T^{-1}]$
  $[h] = M^1L^2T^{-1}$

3. Some special features of dimensions :

- Suppose in any formula, $(L + \alpha)$ term is coming (where L is length). As length can be added only with a length, so α should also be a kind of length.
  So $[\alpha] = [L]$

- Similarly consider a term $(F - \beta)$ where F is force. A force can be added/subtracted with a force only and give rises to a third force. So β should be a kind of force and its result $(F - \beta)$ should also be a kind of force.



Rule No. 1 : ***One quantity can be added / subtracted with a similar quantity only and give rise to the similar quantity.***

Solved Example

Example 1. $\frac{\alpha}{t^2} = Fv + \frac{\beta}{x^2}$

Find dimensional formula for $[\alpha]$ and $[\beta]$ (here t = time, F = force, v = velocity, x = distance)

Solution : Since dimension of $Fv = [Fv] = [M^1L^1T^{-2}]\,[L^1T^{-1}] = [M^1L^2T^{-3}]$,

so $\left[\frac{\beta}{x^2}\right]$ should also be $M^1L^2T^{-3}$

$\frac{[\beta]}{[x^2]} = M^1\,L^2T^{-3}$

$[\beta] = M^1L^4T^{-3}$

and $\left[Fv + \frac{\beta}{x^2}\right]$ will also have dimension $M^1L^2T^{-3}$, so L.H.S. should also have the same dimension $M^1L^2T^{-3}$

so $\frac{[\alpha]}{[t^2]} = M^1L^2T^{-3}$

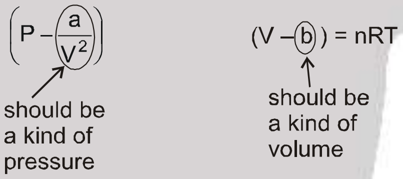
$[\alpha] = M^1L^2T^{-1}$

Example 2. For n moles of gas, Vander waal's equation is

$$\left(P - \frac{a}{V^2}\right)(V - b) = nRT$$

Find the dimensions of a and b, where P is gas pressure, V = volume of gas T = temperature of gas

Solution :



So $\frac{[a]}{[V^2]} = M^1L^{-1}T^{-2}$ So $[b] = L^3$

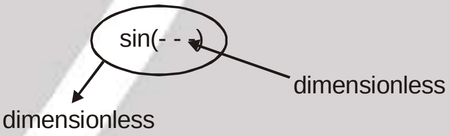
$\frac{[a]}{[L^3]^2} = M^{-1}\,L^{-1}\,T^{-2}$

$\Rightarrow$ $[a] = M^1\,L^5\,T^{-2}$

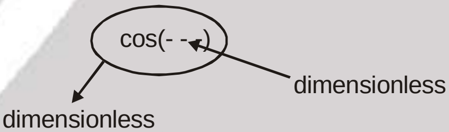
Rule No. 2 : **Consider a term sin(θ)**

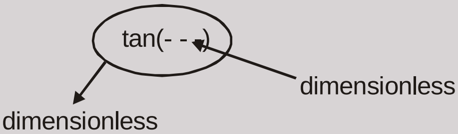
Here θ is dimensionless and sinθ $\left(\frac{\text{Perpendicular}}{\text{Hypoteneous}}\right)$ is also dimensionless.

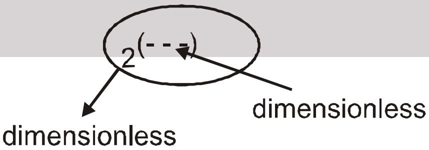
$\Rightarrow$ Whatever comes in sin(......) is dimensionless and entire [sin (.......)] is also dimensionless.

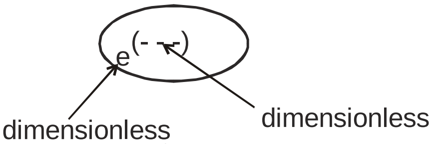


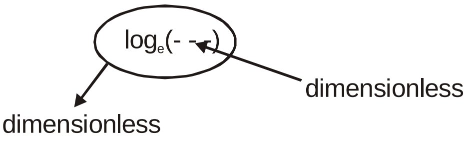
Similarly :











Solved Example

Example 3. $\alpha = \frac{F}{v^2}\sin(\beta t)$ (here v = velocity, F = force, t = time)

Find the dimension of α and β

Solution : $\alpha = \frac{F}{v^2}\sin(\beta t)$

dimensionless

dimensionless

$\Rightarrow [\beta]\ [t] = 1$

$[\beta] = [T^{-1}]$

So $[\alpha] = \frac{[F]}{[v^2]} = \frac{[M^1L^1T^{-2}]}{[L^1T^{-1}]^2} = M^1L^{-1}T^0$

Example 4. $\alpha = \frac{Fv^2}{\beta^2}\log_e\left(\frac{2\pi\beta}{v^2}\right)$ where F = force, v = velocity

Find the dimensions of α and β.

Solution : $\alpha = \frac{Fv^2}{\beta^2}\log_e\frac{2\pi\beta}{v^2}$

dimensionless

dimensionless

$\Rightarrow \frac{[2\pi][\beta]}{[v^2]} = 1 \quad \Rightarrow \quad \frac{[1][\beta]}{L^2T^{-2}} = 1 \quad \Rightarrow \quad [\beta] = L^2T^{-2}$

as $[\alpha] = \frac{[F][v^2]}{[\beta^2]} \quad \Rightarrow \quad [\alpha] = \frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[L^2T^{-2}]^2} \quad \Rightarrow \quad [\alpha] = M^1L^{-1}T^0$

4. USES OF DIMENSIONS :

- **To check the correctness of the formula :**

If the dimensions of the L.H.S and R.H.S are same, then we can say that this eqn. is at least dimensionally correct. So this equation may be correct.

But if dimensions of L.H.S and R.H.S is not same then the equation is not even dimensionally correct. So it cannot be correct.

e.g. A formula is given centrifugal force $F_e = \frac{mv^2}{r}$

(where m = mass , v = velocity , r = radius)

we have to check whether it is correct or not.

Dimension of L.H.S is

$[F] = [M^1L^1T^{-2}]$

Dimension of R.H.S is

$$\frac{[m]\,[v^2]}{[r]} = \frac{[M]\,[LT^{-1}]^2}{[L]} = [M^1L^1T^{-2}]$$

So this eqn. is at least dimensionally correct.

thus we can say that this equation may be correct.

Solved Example

Example 5. Check whether this equation may be correct or not.

Solution : Pressure $P_r = \frac{3Fv^2}{\pi^2t^2x}$ (where P_r = Pressure , F = force , v = velocity, t = time, x = distance)

Dimension of L.H.S = $[P_r] = M^1L^{-1}T^{-2}$

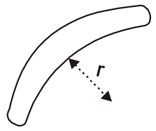
Dimension of R.H.S = $\frac{[3]\,[F]\,[v^2]}{[\pi]\,[t^2]\,[x]} = \frac{[M^1L^1T^{-2}]\,[L^2T^{-2}]}{[T^2]\,[L]} = M^1L^2T^{-6}$

Dimension of L.H.S and R.H.S are not same. So the relation cannot be correct.

Sometimes a question is asked which is beyond our syllabus, then certainly it must be the question of dimensional analyses.

Example 6.

A Boomerang has mass m surface Area A, radius of curvature of lower surface = r and it is moving with velocity v in air of density ρ. The resistive force on it should be –



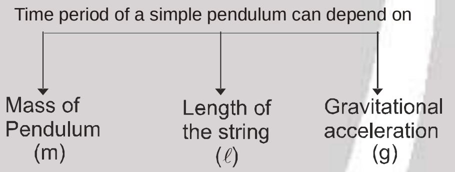
(A) $\frac{2\rho vA}{r^2}\log\left(\frac{\rho m}{\pi Ar}\right)$ (B) $\frac{2\rho v^2A}{r}\log\left(\frac{\rho A}{\pi m}\right)$ (C) $2\rho v^2A\log\left(\frac{\rho Ar}{\pi m}\right)$ (D) $\frac{2\rho v^2A}{r^2}\log\left(\frac{\rho Ar}{\pi m}\right)$

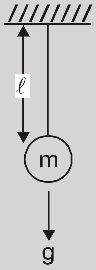
Solution : Only C is dimensionally correct.

- **We can derive a new formula roughly :**
  If a quantity depends on many parameters, we can estimate, to what extent, the quantity depends on the given parameters!

Solved Example

Example 7.





So we can say that expression of T should be in this form

$T = (\text{Some Number})\,(m)^a\,(\ell)^b(g)^c$

Equating the dimensions of LHS and RHS,

$M^0L^0T^1 = (1)\,[M^1]^a\,[L^1]^b\,[L^1T^{-2}]^c$

$M^0L^0T^1 = M^a\,L^{b+c}\,T^{-2c}$

Comparing the powers of M,L and T,

get $a = 0,\ b + c = 0,\ -2c = 1$

so $a = 0,\ b = 1/2,\ c = -1/2$

so $T = (\text{some Number})\,M^0\,L^{1/2}\,g^{-1/2}$

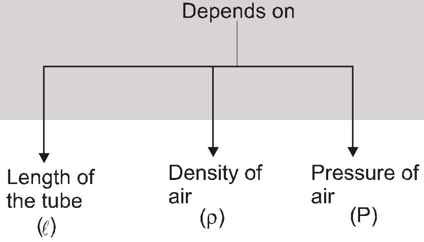
$T = (\text{Some Number})\sqrt{\frac{\ell}{g}}$

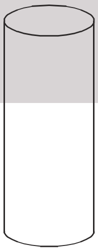
The quantity "Some number" can be found experimentally. Measure the length of a pendulum and oscillate it, find its time period by stopwatch.

Suppose for ℓ = 1m, we get T = 2 sec. so

$2 = (\text{Some Number})\sqrt{\frac{1}{9.8}} \Rightarrow$ "Some number" = 6.28 ≈ 2π.

Example 8. Natural frequency (f) of a closed pipe





So we can say that $f = (\text{some Number})\,(\ell)^a\,(\rho)^b\,(P)^c$

$\left[\frac{1}{T}\right] = (1)\,[L]^a\,[ML^{-3}]^b\,[M^1L^{-1}T^{-2}]^c$

$M^oL^oT^{-1} = M^{b+c}\,L^{a-3b-c}\,T^{-2c}$

Comparing powers of M, L, T

$0 = b + c$

$0 = a - 3b - c$

$-1 = -2c$

get $a = -1$, $b = -1/2$, $c = 1/2$

So $f = \text{(some number)}\ \frac{1}{\ell}\sqrt{\frac{P}{\rho}}$

- **We can express any quantity in terms of the given basic quantities.**

Solved Example

Example 9. If velocity (V), force (F) and time (T) are chosen as fundamental quantities, express (i) mass and (ii) energy in terms of V, F and T

Solution : Let $M = \text{(some Number)}\ (V)^a\,(F)^b\,(T)^c$

Equating dimensions of both the sides

$M^1L^0T^0 = (1)\ [L^1T^{-1}]^a\ [M^1L^1T^{-2}]^b\ [T^1]^c$

$M^1L^0T^0 = M^b\ L^{a+b}\ T^{-a-2b+c}$

get $a = -1$, $b = 1$, $c = 1$

$M = \text{(Some Number)}\ (V^{-1}\,F^1\,T^1) \Rightarrow [M] = [V^{-1}\,F^1\,T^1]$

Similarly we can also express energy in terms of V, F, T

Let $[E] = \text{[some Number]}\ [V]^a\ [F]^b\ [T]^c$

$\Rightarrow$ $[ML^2T^{-2}] = [M^oL^oT^o]\ [LT^{-1}]^a\ [MLT^{-2}]^b\ [T]^c$

$\Rightarrow$ $[M^1L^2T^{-2}] = [M^b\ L^{a+b}\ T^{-a-2b+c}] \Rightarrow$ $1 = b$; $2 = a + b$; $-2 = -a - 2b + c$

get $a = 1$; $b = 1$; $c = 1$

$\therefore$ $E = \text{(some Number)}\ V^1F^1T^1$ or $[E] = [V^1][F^1][T^1]$.

- **To find out unit of a physical quantity :**

  Suppose we want to find the unit of force. We have studied that the dimension of force is
  $[\text{Force}] = [M^1L^1T^{-2}]$

  As unit of M is kilogram (kg) , unit of L is meter (m) and unit of T is second (s) so unit of force can be written as $(kg)^1\ (m)^1\ (s)^{-2}$ = kg m/s^2 in MKS system. In CGS system, unit of force can be written as $(g)^1\ (cm)^1\ (s)^{-2}$ = g cm/s^2.

LIMITATIONS OF DIMENSIONAL ANALYSIS :

From Dimensional analysis we get $T = \text{(Some Number)}\ \sqrt{\frac{\ell}{g}}$

so the expression of T can be

$T = 2\sqrt{\frac{\ell}{g}}$ $T = \sqrt{\frac{\ell}{g}}\ \sin(.....)$

or or

$T = 50\sqrt{\frac{\ell}{g}}$ $T = \sqrt{\frac{\ell}{g}}\ \log(......)$

or or

$T = 2\pi\sqrt{\frac{\ell}{g}}$ $T = \sqrt{\frac{\ell}{g}} + (t_0)$

- Dimensional analysis doesn't give information about the "some Number" : The dimensional constant.
- This method is useful only when a physical quantity depends on other quantities by multiplication and power relations.

  (i.e., $f = x^a\ y^b\ z^c$)

  It fails if a physical quantity depends on sun or difference of two quantities

  (i.e.$f = x + y - z$)

  i.e., we cannot get the relation $S = ut + \frac{1}{2}at^2$ from dimensional analysis.

- This method will not work if a quantity depends on another quantity as sine or cosine, logarithmic or exponential relation. The method works only if the dependence is by power functions.
- We equate the powers of M, L and T hence we get only three equations. So we can have only three variable (only three dependent quantities)

  So dimensional analysis will work only if the quantity depends only on three parameters, not more than that.

Solved Example

Example 10. Can Pressure (P), density (ρ) and velocity (v) be taken as fundamental quantities ?

Solution : P, ρ and v are not independent, they can be related as $P = \rho v^2$,so they cannot be taken as fundamental variables.

To check whether the 'P', 'ρ', and 'V' are dependent or not, we can also use the following mathematical method :

$[P] = [M^1L^{-1}T^{-2}]$

$[\rho] = [M^1L^{-3}\,T^0]$

$[V] = [M^0L^1T^{-1}]$

Check the determinant of their powers :

$$\begin{vmatrix} 1 & -1 & -2 \\ 1 & -3 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 1\,(3) - (-1)(-1) - 2\,(1) = 0,$$

So these three terms are dependent.

UNIT :

- **Unit :**

  Measurement of any physical quantity is expressed in terms of an internationally accepted certain basic standard called unit.
- **SI Units :**

  In 1971, an international Organization "CGPM" : (General Conference on weight and Measure) decided the standard units, which are internationally accepted. These units are called SI units (International system of units)

1. SI Units of Basic Quantities :

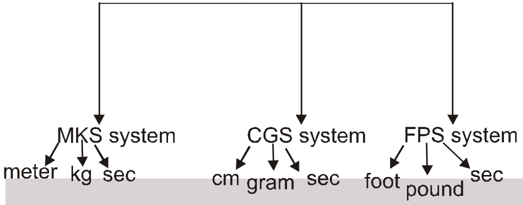
Base Quantity	SI Units		
	Name	Symbol	Definition
Length	metre	m	The metre is the length of the path traveled by light in vacuum during a time interval of 1/299, 792, 458 of a second (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967)
Electric Current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, will produce between these conductors a force equal to 2 x 10^{-7} Newton per metre of length. (1948)
Thermodynamic Temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of Substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)

2. Two supplementary units were also defined :

- Plane angle – Unit = radian (rad)
- Solid angle – Unit = Steradian (sr)

3. Other classification :

If a quantity involves only length, mass and time (quantities in mechanics), then its unit can be written in MKS, CGS or FPS system.



- **For MKS system :**
  In this system Length, mass and time are expressed in meter, kg and second. respectively. It comes under SI system.
- **For CGS system :**
  In this system ,Length, mass and time are expressed in cm, gram and second. respectively.
- **For FPS system :**
  In this system, length, mass and time are measured in foot, pound and second. respectively.

4. SI units of derived Quantities :

- Velocity = $\frac{\text{displacement(metre)}}{\text{time(second)}}$
  So unit of velocity will be m/s
- Acceleration = $\frac{\text{change in velocity}}{\text{time}} = \frac{m/s}{s} = \frac{m}{s^2}$
- Momentum = mv
  so unit of momentum will be = (kg) (m/s) = kg m/s
- Force = ma
  Unit will be = (kg) × (m/s^2) = kg m/s^2 called newton (N)
- Work = FS
  unit = (N) × (m) = N m called joule (J)
- Power = $\frac{\text{work}}{\text{time}}$
  Unit = J / s called watt (W)

5. Units of some physical Constants :

- Unit of "Universal Gravitational Constant" (G)

$$F = \frac{G(m_1)(m_2)}{r^2} \quad \Rightarrow \quad \frac{kg \times m}{s^2} = \frac{G(kg)(kg)}{m^2}$$

so unit of G = $\frac{m^3}{kg\, s^2}$

6. SI Prefix :

Suppose distance between kota to Jaipur is 3000 m. so

$$d = 3000 \text{ m} = 3 \times \underset{\text{kilo(k)}}{\underset{\downarrow}{\boxed{1000}}}\text{m}$$

= 3 km (here 'k' is the prefix used for 1000 (10^3))

Suppose thickness of a wire is 0.05 m

$$d = 0.05 \text{ m} = 5 \times \underset{\text{centi(c)}}{\underset{\downarrow}{\boxed{10^{-2}}}}\text{m}$$

= 5 cm (here 'c' is the prefix used for (10^{-2}))

Similarly, the magnitude of physical quantities vary over a wide range. So in order to express the very large magnitude as well as very small magnitude more compactly, "CGPM" recommended some standard prefixes for certain power of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	K	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

Solved Example

Example 11. Convert all in meters (m) :

(i) 5 μm. (ii) 3 km (iii) 20 mm (iv) 73 pm (v) 7.5 nm

Solution :

(i) 5 μm $= 5 \times 10^{-6}$m
(ii) 3 km $= 3 \times 10^{3}$ m
(iii) 20 mm $= 20 \times 10^{-3}$m
(iv) 73 pm $= 73 \times 10^{-12}$ m
(v) 7.5 nm $= 7.5 \times 10^{-9}$ m

Example 12. F = 5 N convert it into CGS system.

Solution :

$$F = 5\frac{kg \times m}{s^2} = (5)\ \frac{(10^3 g)(100\,cm)}{s^2} = 5 \times 10^5\ \frac{g\,cm}{s^2}$$ (in CGS system).

This unit ($\frac{g\,cm}{s^2}$) is also called dyne

Example 13. $G = 6.67 \times 10^{-11}\ \frac{m^3}{kg\,s^2}$ convert it into CGS system.

Solution : $G = 6.67 \times 10^{-11}\ \frac{m^3}{kg\,s^2}$

$$= (6.67\times10^{-11})\ \frac{(100\,cm)^3}{(1000\,g)s^2} = 6.67 \times 10^{-8}\ \frac{cm^3}{g s^2}$$

Example 14. $\rho = 2$ g/cm^3

convert it into MKS system.

Solution : $\rho = 2$ g/cm^3

$$= (2)\ \frac{10^{-3}\,kg}{(10^{-2}m)^3}$$

$= 2 \times 10^3$ kg/m^3

Example 15. V = 90 km / hour

convert it into m/s.

Solution : V = 90 km / hour $= (90)\ \frac{(1000m)}{(60 \times 60\,second)}$

$$V = (90)\left(\frac{1000}{3600}\right)\frac{m}{s}$$

$$V = 90 \times \frac{5}{18}\ \frac{m}{s}$$

V = 25 m/s

POINT TO REMEMBER :

To convert km/hour into m/sec, multiply by 5/18.

Solved Example

Example 16. Convert 7 pm into μm.

Sol. Let $7\text{ pm} = (x)\ \mu\text{m}$, Now lets convert both LHS & RHS into meter

$7 \times (10^{-12})\text{ m} = (x) \times 10^{-6}\text{ m}$

get $x = 7 \times 10^{-6}$

So $7\text{ pm} = (7\times10^{-6})\ \mu\text{m}$

Some SI units of derived quantities are named after the scientist, who has contributed in that field a lot.

8. SI Derived units, named after the scientist :

S.N	Physical Quantity	SI Units			
		Unit name	Symbol of the unit	Expression in terms of other units	Expression in terms of base units
1.	Frequency $(f = \frac{1}{T})$	hertz	Hz	$\frac{\text{Oscillation}}{\text{s}}$	s^{-1}
2.	Force (F = ma)	newton	N	- - - - -	$Kg\ m / s^2$
3.	Energy, Work, Heat (W = Fs)	joule	J	Nm	$Kg\ m^2 / s^2$
4.	Pressure, stress $(P = \frac{F}{A})$	pascal	Pa	N / m^2	$Kg / m\ s^2$
5.	Power, $(\text{Power} = \frac{W}{t})$	watt	W	J / s	$Kg\ m^2 / s^3$

CHANGE OF NUMERICAL VALUE WITH THE CHANGE OF UNIT :

Suppose we have

$$\ell = 7\text{ cm} \xrightarrow[\text{it into metres, we get}]{\text{If we convert}} = \frac{7}{100}\text{m}$$

we can say that if the unit is increased to 100 times (cmm),

the numerical value became $\frac{1}{100}$ times $\left(7 \rightarrow \frac{7}{100}\right)$

So we can say

Numerical value $\propto \frac{1}{\text{unit}}$

We can also tell it in a formal way like the following :

Magnitude of a physical quantity = (Its Numerical value) (unit)

= (n) (u)

Magnitude of a physical quantity always remains constant, it will not change if we express it in some other unit.

$$\begin{tikzcd} & \text{So } (n)\ (u) = \text{constant} \arrow[dl] \arrow[dr] & \\ n \propto \frac{1}{u} & & n_1u_1 = n_2u_2 \end{tikzcd}$$

numerical value $\propto \frac{1}{\text{unit}}$

Solved Example

Example 17. If unit of length is doubled, the numerical value of Area will be ....

Solution : As unit of length is doubled, unit of Area will become four times. So the numerical value of Area will became one fourth. Because numerical value $\propto \frac{1}{\text{unit}}$,

Example 18. Force acting on a particle is 5N.If unit of length and time are doubled and unit of mass is halved than the numerical value of the force in the new unit will be.

Solution : Force = 5 $\frac{\text{kg} \times \text{m}}{\text{sec}^2}$

If unit of length and time are doubled and the unit of mass is halved.

Then the unit of force will be $\left(\frac{\frac{1}{2} \times 2}{(2)^2} \right) = \frac{1}{4}$ times

Hence the numerical value of the force will be 4 times. (As numerical value $\propto \frac{1}{\text{unit}}$)

EXERCISE

1. Which of the following sets cannot enter into the list of fundamental quantities in any system of units?

(A) length, mass and velocity (B) length, time and velocity
(C) mass, time and velocity (D) length, time and mass

2. A dimensionless quantity

(A) never has a unit (B) always has a unit (C) may have a unit (D) does not exit

3. A unit less quantity

(A) never has a nonzero dimension (B) always has a nonzero dimension
(C) may have a nonzero dimension (D) does not exit

4. The velocity of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. The method of dimensions gives the relation between these quantities as

(A) $v^2 = k\lambda^{-1} g^{-1} \rho^{-1}$ (B) $v^2 = k\, g\, \lambda$ (C) $v^2 = k\, g\, \lambda\, \rho$ (D) $v^2 = k\, \lambda^3 g^{-1} \rho^{-1}$
where k is a dimensionless constant

5. The value of $G = 6.67 \times 10^{-11}$ N m^2 (kg)$^{-2}$. Its numerical value in CGS system will be :

(A) 6.67×10^{-8} (B) 6.67×10^{-6} (C) 6.67 (D) 6.67×10^{-5}

6. Force applied by water stream depends on density of water (ρ), velocity of the stream (v) and cross–sectional area of the stream (A). The expression of the force should be

(A) $\rho A v$ (B) $\rho A v^2$ (C) $\rho^2 A v$ (D) $\rho A^2 v$

7. If unit of length and time is doubled, the numerical value of 'g' (acceleration due to gravity) will be :

(A) doubled (B) halved (C) four times (D) remain same

8. Force F is given in terms of time t and distance x by

$$F = A \sin C t + B \cos D x$$

Then the dimensions of $\frac{A}{B}$ and $\frac{C}{D}$ are given by

(A) $MLT^{-2}, M^0L^0T^{-1}$ (B) $MLT^{-2}, M^0L^{-1}T^0$ (C) $M^0L^0T^0, M^0L^1T^{-1}$ (D) $M^0L^1T^{-1}, M^0L^0T^0$

9.* Choose the correct statement(s):

(A) All quantities may be represented dimensionally in terms of the base quantities.

(B) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.

(C) The dimension of a base quantity in other base quantities is always zero.

(D) The dimension of a derived quantity is never zero in any base quantity.

10.* Choose the correct statement(s) :

(A) A dimensionally correct equation may be correct.

(B) A dimensionally correct equation may be incorrect.

(C) A dimensionally incorrect equation may be correct.

(D) A dimensionally incorrect equation must be incorrect.

11. In the formula; $p = \frac{nRT}{V-b} e^{-\frac{a}{RTV}}$, find the dimensions of 'a' and 'b', where p = pressure, n = no. of moles , T = temperature , V = volume and R = universal gas constant .

Comprehension # 1

The Vander waal equation for 1 mole of a real gas is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are Vander waal constants.

12. The dimensions of a are the same as those of

(A) PV (B) PV^2 (C) P^2V (D) P/V

13. The dimensions of b are the same as those of

(A) P (B) V (C) PV (D) nRT

14. The dimensional formula for ab is

(A) $ML^2\,T^{-2}$ (B) $ML^4\,T^{-2}$ (C) $ML^6\,T^{-2}$ (D) $ML^8\,T^{-2}$

ANSWER KEY OF EXERCISE

1.	(B)	**2.**	(C)	**3.**	(A)	**4.**	(B)	**5.**	(A)	**6.**	(B)	**7.**	(A)
8.	(C)	**9.***	(A) (B) (C)	**10.***	(A) (B) (D)	**11.**	$ML^5\,T^{-2}\,mol^{-1}$.	**12.**	(B)				
13.	(B)	**14.**	(D)										